Fluctuation-Dissipation Relations for a Nonlocal Plasma

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A generalized version of the Callen-Welton fluctuation-dissipation formula that is nonlocal in space and time is derived. In a nonuniform plasma there appear significant differences between the fluctuations of the electrostatic field and those of the electron density, and the spatial inhomogeneities lead to an asymmetry with respect to the sign inversion of the frequency.

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The study of fluctuations attracts a great deal of attention. Besides being of interest from a fundamental point of view, there are situations for which nonequilibrium fluctuations play an important role, namely, in the neighborhood of bifurcations where the system has to choose a branch [1]. Moreover fluctuations find applications in diagnostic procedures. Indeed, plasma parameters such as temperature, mean velocity, density, and their respective profiles can be determined from incoherent (Thomson) scattering diagnostics [2], i.e., by the proper interpretation of the data obtained from the scattering of a given electromagnetic field interacting with the system. The key point for this interpretation is the knowledge of the intensity of the dielectric function fluctuations or equivalently of the electron form factor. The electron form factor and the electrostatic field fluctuations have been the object of active investigation since the early 1960s [2]. In thermodynamic equilibrium, the electrostatic field fluctuations satisfy the famous Callen-Welton fluctuation-dissipation theorem [3], linking their intensity to the *imaginary* part of the dielectric function and to the temperature. The matter becomes more delicate in the local-equilibrium case. We have indeed shown that in the *collisional regime* the Callen-Welton formula should be revised [4]. There then appear new terms explicitly displaying dissipative nonequilibrium contributions and containing the interparticle collision frequency, the differences in the temperatures and the velocities, and also functions of the *real* parts of the dielectric susceptibilities. However, it is not evident that the plasma parameters can be kept *constant* in both space and time. Inhomogeneities in space and time of these quantities will certainly also contribute to the fluctuations. In this Letter, using the Langevin approach and the time-space multiscale technique, we show that not only the *imaginary* part but also the derivatives of the *real* part of the dielectric susceptibility determine the amplitude and the width of the spectral lines of the electrostatic field fluctuations, as well as the form factor. As a result of the inhomogeneity, these properties become asymmetric with respect to the inversion of the sign of the frequency. In the kinetic regime the form factor is more sensitive to space gradients than the spectral function of the electrostatic field fluctuations. Note that for simple fluids and gases a general theory of hydrodynamic fluctuations for nonequilibrium stationary inhomogeneous states has been developed in a series of publications [5]. In particular, it has been found that there exists an asymmetry of the spectrum for Brillouin scattering from a fluid in a shear flow or in a temperature gradient. The situation for the plasma problem we are considering is, however, quite different.

To derive nonlocal expressions for the spectral function of the electrostatic field fluctuation and for the electron form factor, we use the Langevin approach to describe kinetic fluctuations [6]. The starting point of our procedure is the same as in [4]. A kinetic equation for the fluctuation δf_a of the one-particle distribution function (DF) with respect to the reference state f_a is considered. In the general case the reference state is a nonequilibrium DF which varies in space and time both on the kinetic scale (mean-free path l_{ei} and interparticle collision time ν_{ei}^{-1}) and on the larger hydrodynamic scales. These scales are much larger than the characteristic fluctuation time ω^{-1}. In the nonequilibrium case we can therefore introduce a small parameter $\mu = \nu_{ei}/\omega$, which allows us to describe fluctuations on the basis of a multiple space and time scale analysis. Obviously, the fluctuations vary on both the "fast" $(\mathbf{r}, t)$ and the "slow" $(\mu\mathbf{r}, \mu t)$ time and space scales: $\delta f_a(\mathbf{x}, t) = \delta f_a(\mathbf{x}, t, \mu t, \mu\mathbf{r})$ and $f_a(\mathbf{x}, t) = f_a(\mathbf{p}, \mu t, \mu\mathbf{r})$. Here $\mathbf{x}$ stands for the phase-space coordinates $(\mathbf{r}, \mathbf{p})$. The Langevin kinetic equation for δf_a has the form [4,6]

$$\hat{L}_{a\mathbf{x}t}[\delta f_a(\mathbf{x}, t) - \delta f_a^S(\mathbf{x}, t)] = -e_a \delta\mathbf{E}(\mathbf{r}, t) \cdot \frac{\partial f_a(\mathbf{x}, t)}{\partial \mathbf{p}}, \tag{1}$$

where e_a is the charge of the particle of specie a, $\delta\mathbf{E}$ is the electrostatic field fluctuation, and the operator $\hat{L}_{a\mathbf{x}t}$ is defined by $\hat{L}_{a\mathbf{x}t} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} + \hat{\Gamma}_a(\mathbf{x},t)$; $\hat{\Gamma}_a(\mathbf{x}, t) = e_a\mathbf{E} \cdot \frac{\partial}{\partial \mathbf{p}} - \delta\hat{I}_a$, and $\delta\hat{I}_a$ is the linearized collision operator. The Langevin source δf_a^S in Eq. (1) is determined by the following equation [4,6]: $\hat{L}_{a\mathbf{x}t}\overline{\delta f_a(\mathbf{x}, t)\delta f_b(\mathbf{x}', t')}^S = \delta_{ab}\delta(t - t')\delta(\mathbf{x} - \mathbf{x}')f_b(\mathbf{x}', t')$. The solution of Eq. (1)

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has the form

$$\delta f_a(\mathbf{x},t) = \delta f_a^S(\mathbf{x},t) - \sum_b \int d\mathbf{x}' \int_{-\infty}^{t} dt' \times G_{ab}(\mathbf{x},t,\mathbf{x}',t') e_b \delta\mathbf{E}(\mathbf{r}',t') \cdot \frac{\partial f_b(\mathbf{x}',t')}{\partial \mathbf{p}'}, \tag{2}$$

where the Green function $G_{ab}(\mathbf{x},t,\mathbf{x}',t')$ of the operator $\hat{L}_{a\mathbf{x}t}$ is determined by $\hat{L}_{a\mathbf{x}t} G_{ab}(\mathbf{x},t,\mathbf{x}',t') = \delta_{ab}\delta(\mathbf{x}-\mathbf{x}') \times \delta(t-t')$ with the causality condition: $G_{ab}(\mathbf{x},t,\mathbf{x}',t') = 0$, when $t < t'$. Thus, $\overline{\delta f_a(\mathbf{x},t)\delta f_b(\mathbf{x}',t')}^S$ and $G_{ab}(\mathbf{x},t,\mathbf{x}',t')$ are connected by the relation: $\overline{\delta f_a(\mathbf{x},t)\delta f_b(\mathbf{x}',t')}^S = G_{ab}(\mathbf{x},t,\mathbf{x}',t') f_b(\mathbf{x}',t')$, $t > t'$. For stationary and spatially uniform systems the DF f_a and the operator $\hat{\Gamma}_a$ do not depend on time and space. In this case, the dependence on time and space of the Green function $G_{ab}(\mathbf{x},t,\mathbf{x}',t')$ appears only through the difference $t - t'$ and $\mathbf{r} - \mathbf{r}'$. However, when the DF $f_a(\mathbf{p},\mu\mathbf{r},\mu t)$ and $\hat{\Gamma}_a(\mathbf{p},\mu\mathbf{r},\mu t)$ are slowly varying quantities in time and space, and when nonlocal effects are considered, the time and space dependence of $G_{ab}(\mathbf{x},t,\mathbf{x}',t')$ is more subtle:

$$G_{ab}(\mathbf{x},t,\mathbf{x}',t') = G_{ab}(\mathbf{p},\mathbf{p}',\mathbf{r}-\mathbf{r}',t-t',\mu\mathbf{r}',\mu t'). \tag{3}$$

For the homogeneous case this nontrivial result was obtained for the first time in our previous work [7]. Recently, this result was extended to inhomogeneous systems [8]. Here we want to stress that the nonlocal effects appear due to the slow time and space dependences $\mu\mathbf{r}'$ and $\mu t'$.

At first order, the expansion with respect to μ, Eq. (2) leads to

$$\delta f_a(\mathbf{x},t) = \delta f_a^S(\mathbf{x},t) - \sum_b e_b \int d\mathbf{p}' d\boldsymbol{\rho} \int_0^{\infty} d\tau \left(1 - \mu\tau \frac{\partial}{\partial \mu t} - \mu\boldsymbol{\rho} \cdot \frac{\partial}{\partial \mu\mathbf{r}}\right) \times G_{ab}(\boldsymbol{\rho},\tau,\mathbf{p},\mathbf{p}',\mu t,\mu\mathbf{r}) \delta\mathbf{E}(\mathbf{r}-\boldsymbol{\rho},t-\tau) \cdot \frac{\partial f_b(\mathbf{p}',\mu t,\mu\mathbf{r})}{\partial \mathbf{p}'}, \tag{4}$$

with $\boldsymbol{\rho} = \mathbf{r} - \mathbf{r}'$ and $\tau = t - t'$.

From the Poisson equation

$$\delta\mathbf{E}(\mathbf{r},t) = -\frac{\partial}{\partial \mathbf{r}} \sum_b e_b \int \frac{1}{|\mathbf{r}-\mathbf{r}'|} \delta f_b(\mathbf{x}',t) \, d\mathbf{x}', \tag{5}$$

and performing the Fourier-Laplace transformation for the fast variables $\delta\mathbf{E}(\mathbf{k},\omega) = \int_0^\infty dt \int d\mathbf{r}\, \delta\mathbf{E}(\mathbf{r},t) \exp(-\Delta t + i\omega t - i\mathbf{k}\cdot\mathbf{r})$, we have

$$\delta\mathbf{E}(\mathbf{k},\omega,t,\mathbf{r}) = \delta\mathbf{E}^s(\mathbf{k},\omega) + \sum_a 4\pi i e_a^2 \int d\mathbf{p} \left(1 + i\frac{\partial}{\partial\omega}\frac{\partial}{\partial t} - i\frac{\partial}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}}\right) \frac{\mathbf{k}}{k^2} \hat{L}_{a\omega\mathbf{k}}^{-1} \delta\mathbf{E}(\mathbf{k},\omega,\mathbf{r},t) \cdot \frac{\partial f_a(\mathbf{p},\mathbf{r},t)}{\partial \mathbf{p}}. \tag{6}$$

Here and in the following for simplicity we omit μ, keeping in mind that derivatives over coordinates and time are taken with respect to the slowly varying variables. The resolvent $\hat{L}_{a\omega\mathbf{k}}^{-1}$ in Eq. (6) is determined by the following relation: $\hat{L}_{a\omega\mathbf{k}}^{-1}\delta_{ab}\delta(\mathbf{p}-\mathbf{p}') = \int d\boldsymbol{\rho} \int_0^\infty d\tau \exp(-\Delta\tau + i\omega\tau - i\mathbf{k}\cdot\boldsymbol{\rho}) G_{ab}(\boldsymbol{\rho},\tau,\mathbf{p},\mathbf{p}',\mu t,\mu\mathbf{r})$. One should bear in mind that the derivatives $\frac{\partial}{\partial\omega}$ and $\frac{\partial}{\partial\mathbf{k}}$ in Eq. (6) act only on the operator $\frac{\mathbf{k}}{k^2}\hat{L}_{a\omega\mathbf{k}}^{-1}$. The approximation in which Eq. (6) was derived corresponds to the geometric optics approximation [9]. At first order and after some manipulations, one obtains from Eq. (6) the transport equation in the geometric optics approximation, which is not considered in the Letter, and the equation for the spectral function of the electrostatic field fluctuations:

$$\mathrm{Re}\varepsilon(\omega,\mathbf{k}) \left[(\delta\mathbf{E}\delta\mathbf{E})_{\omega,\mathbf{k}} - \frac{1}{|\tilde{\varepsilon}(\omega,\mathbf{k})|^2} (\delta\mathbf{E}\delta\mathbf{E})_{\omega,\mathbf{k}}^S\right] = 0, \tag{7}$$

where we introduced $\tilde{\varepsilon}(\omega,\mathbf{k}) = 1 + \sum_a \tilde{\chi}_a(\omega,\mathbf{k})$, $\varepsilon(\omega,\mathbf{k}) = 1 + \sum_a \chi_a(\omega,\mathbf{k})$,

$$\tilde{\chi}_a(\omega,\mathbf{k}) = \left(1 + i\frac{\partial}{\partial\omega}\frac{\partial}{\partial t} - i\frac{\partial}{\partial\mathbf{r}}\cdot\frac{\partial}{\partial\mathbf{k}}\right) \chi_a(\omega,\mathbf{k},t,\mathbf{r}), \tag{8}$$

and where $\chi_a(\omega,\mathbf{k},t,\mathbf{r}) = -\frac{4\pi i e_a^2}{k^2} \int d\mathbf{p}\, \hat{L}_{a\omega\mathbf{k}}^{-1} \mathbf{k} \cdot \frac{\partial}{\partial\mathbf{p}} \times f_a(\mathbf{p},t,\mathbf{r})$ is the susceptibility for a collisional plasma. In the same approximation the spectral function of the Langevin source $(\delta\mathbf{E}\delta\mathbf{E})_{\omega,\mathbf{k}}^S$ takes the form

$$(\delta\mathbf{E}\delta\mathbf{E})_{\omega,\mathbf{k}}^S = 32\pi^2 \sum_a e_a^2 \mathrm{Re} \int d\mathbf{p} \times \left(1 + i\frac{\partial}{\partial\omega}\frac{\partial}{\partial t} - i\frac{\partial}{\partial\mathbf{k}}\cdot\frac{\partial}{\partial\mathbf{r}}\right) \times \frac{1}{k^2} \hat{L}_{a\omega\mathbf{k}}^{-1} f_a(\mathbf{p},\mathbf{r},t). \tag{9}$$

If $\mathrm{Re}\boldsymbol{\varepsilon}(\omega,\mathbf{k}) \neq 0$, it follows from Eqs. (7) and (9) that the spectral function of the nonequilibrium electrostatic field fluctuations is determined by the expression

$$(\delta\mathbf{E}\delta\mathbf{E})_{\omega,\mathbf{k}} = \frac{32\pi^2 \sum_a e_a^2 \mathrm{Re} \int d\mathbf{p}\, (1 + i\frac{\partial}{\partial\omega}\frac{\partial}{\partial t} - i\frac{\partial}{\partial\mathbf{k}}\cdot\frac{\partial}{\partial\mathbf{r}}) \frac{1}{k^2} \hat{L}_{a\omega\mathbf{k}}^{-1} f_a(\mathbf{p},\mathbf{r},t)}{|\tilde{\varepsilon}(\omega,\mathbf{k})|^2}. \tag{10}$$

The effective dielectric function $\tilde{\varepsilon}(\omega, \mathbf{k})$ in the denominator of Eq. (10) determines the spectral properties of the electrostatic field fluctuations and its imaginary part

$$\mathrm{Im}\tilde{\varepsilon}(\omega, \mathbf{k}) = \mathrm{Im}\varepsilon(\omega, \mathbf{k}) + \frac{\partial}{\partial \omega}\frac{\partial}{\partial t}\mathrm{Re}\varepsilon(\omega, \mathbf{k}, t, \mathbf{r}) - \frac{\partial}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}}\mathrm{Re}\varepsilon(\omega, \mathbf{k}, t, \mathbf{r}) \tag{11}$$

determines the width of the spectral lines near the resonance. Note that when expanding the Green function in Eq. (4) in terms of the small parameter μ, there appear additional terms at first order. It is important to note that the *imaginary* part of the dielectric susceptibility is now replaced by the *real* part, which is greater than the *imaginary* part by the factor μ^{-1}. Therefore, the second and third terms in Eq. (11) in the kinetic regime have an effect comparable to that of the first term. At second order in the expansion in μ the corrections appear only in the *imaginary* part of the susceptibility, and they can reasonably be neglected. It is therefore sufficient to retain the first order corrections to solve the problem.

For the local equilibrium case where the reference state f_a is Maxwellian, we have the identity, $\int d\mathbf{p} \times (1 + i\frac{\partial}{\partial \omega}\frac{\partial}{\partial t} - i\frac{\partial}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}})\frac{1}{k^2}\hat{L}^{-1}_{a\omega\mathbf{k}} f_a(\mathbf{p}, t, \mathbf{r}) = \frac{i}{\omega_a}\int d\mathbf{p} \times f_a(\mathbf{p}, t, \mathbf{r}) - \frac{i\Theta_a}{\omega_a 4\pi e_a^2}\tilde{\chi}_a(\omega, \mathbf{k})$ ($\omega_a = \omega - \mathbf{k}\mathbf{V}_a$, and Θ is the temperature in energy units), and Eq. (10) takes the form

$$(\delta\mathbf{E}\delta\mathbf{E})_{\omega,\mathbf{k}} = \sum_a \frac{8\pi\Theta_a}{\omega_a|\tilde{\varepsilon}(\omega, \mathbf{k})|^2}\mathrm{Im}\tilde{\chi}_a(\omega, \mathbf{k}). \tag{12}$$

In this case the small parameter μ is determined on the slower hydrodynamic scale. For the case of equal temperatures and $\mathbf{V}_a = 0$, one obtains a generalized expression for the Callen-Welton formula:

$$(\delta\mathbf{E}\delta\mathbf{E})_{\omega,\mathbf{k}} = \frac{8\pi\Theta\,\mathrm{Im}\tilde{\varepsilon}(\omega, \mathbf{k})}{\omega|\tilde{\varepsilon}(\omega, \mathbf{k})|^2}. \tag{13}$$

To calculate explicitly $(\delta\mathbf{E}\delta\mathbf{E})_{\omega,\mathbf{k}}$ we will restrict our analysis to the vicinity of the resonance, i.e., $\omega = \pm\omega_0$, where $\mathrm{Re}\varepsilon(\omega_0, \mathbf{k}) = 0$. We can develop $\tilde{\varepsilon}(\omega, \mathbf{k}) = (\omega - \omega_0\,\mathrm{sgn}\omega)\frac{\partial\,\mathrm{Re}\varepsilon}{\partial\omega}\rfloor_{\omega=\omega_0\,\mathrm{sgn}\,\omega} + i[\mathrm{Im}\varepsilon + (\frac{\partial^2}{\partial\omega\,\partial t} - \frac{\partial}{\partial\mathbf{k}} \cdot \frac{\partial}{\partial\mathbf{r}}) \times \mathrm{Re}\varepsilon]\rfloor_{\omega=\omega_0\,\mathrm{sgn}\,\omega}$. Thus $(\delta\mathbf{E}\delta\mathbf{E})_{\omega,\mathbf{k}} = \frac{\tilde{\gamma}}{(\omega-\omega_0\,\mathrm{sgn}\,\omega)^2+\tilde{\gamma}^2} \times \frac{8\pi T}{\omega\,\partial\,\mathrm{Re}\varepsilon/\partial\omega}\rfloor_{\omega=\omega_0}$, where

$$\tilde{\gamma} = \left[\mathrm{Im}\varepsilon + \frac{\partial^2}{\partial\omega\,\partial t}\mathrm{Re}\varepsilon - \frac{\partial}{\partial\mathbf{k}} \cdot \frac{\partial}{\partial\mathbf{r}}\mathrm{Re}\varepsilon\right] \Big/ \frac{\partial\,\mathrm{Re}\varepsilon}{\partial\omega}\Big\rfloor_{\omega=\omega_0\,\mathrm{sgn}\,\omega} \tag{14}$$

is the effective damping decrement. For the case where the system parameters are homogeneous in space but vary in time, the correction is still symmetric with respect to the change of sign of ω, but the intensities and broadening are different, and the intensity integrated over the frequencies remains the same as in the stationary case. However, when the plasma parameters are space dependent this symmetry is lost. In the same manner as for simple fluids and gases [5] the spectral asymmetry is related to the appearance of space anisotropy in inhomogeneous systems. The *real* part of the susceptibility $\mathrm{Re}\varepsilon$ is an even function of ω. This property implies that the contribution of the third term to the expression of the damping decrement (14) is an odd function of ω. Moreover this term gives rise to an anisotropy in k space. Let us estimate this correction for the plasma mode $(\omega_0 = \omega_L)\,\mathrm{Re}\varepsilon = 1 - \frac{\omega_L^2}{\omega^2}(1 + 3\frac{k^2\Theta}{m\omega^2})$, $\mathrm{Im}\varepsilon = \frac{\omega_L^2}{\omega^2}\frac{\nu_{ei}}{\omega}$, $\omega_L^2 = \frac{4\pi n e^2}{m} = \frac{\Theta k_D^2}{m}$, and

$$\tilde{\gamma} = \left[\nu_{ei} + \frac{2}{n}\frac{\partial n}{\partial t} + 6\frac{\omega_L}{nk_D^2}\mathbf{k} \cdot \frac{\partial n}{\partial \mathbf{r}}\,\mathrm{sgn}\omega\right] \Big/ 2. \tag{15}$$

On the hydrodynamic scale $|\frac{2}{n}\frac{\partial n}{\partial t}|, |\frac{6\omega_L}{nk_D^2}\mathbf{k} \cdot \frac{\partial n}{\partial \mathbf{r}}| < \nu_{ei}$, and $\tilde{\gamma} > 0$.

For the spatially homogeneous case there is no difference between the spectral properties of the longitudinal electric field and of the electron density. They are connected by the Poisson equation. This statement is no longer valid when considering an inhomogeneous plasma. Indeed the longitudinal electric field is linked to the particle density by the nonlocal Poisson relation (5). In the latter case, an analysis similar to that made above can also be performed for the particle density. From Eq. (2) there follows

$$\delta n_a(\mathbf{k}, \omega, \mathbf{r}, t) = \delta n_a^S(\mathbf{k}, \omega, \mathbf{r}, t) + \sum_b \frac{4\pi i\mathbf{k} e_b e_a}{k^2}\int d\mathbf{p}\left(1 + i\frac{\partial}{\partial\omega}\frac{\partial}{\partial t} - i\frac{\partial}{\partial\mathbf{k}} \cdot \frac{\partial}{\partial\mathbf{r}}\right)\hat{L}^{-1}_{a\omega\mathbf{k}}\delta n_b(\mathbf{k}, \omega, \mathbf{r}, t) \cdot \frac{\partial f_a(\mathbf{p}, r, t)}{\partial\mathbf{p}}. \tag{16}$$

One should remember that now the derivatives $\frac{\partial}{\partial\omega}$ and $\frac{\partial}{\partial\mathbf{k}}$ in Eq. (16) act only on the operator $\hat{L}^{-1}_{a\omega\mathbf{k}}$. At the first order approximation and after some manipulations, one obtains the following expression for the electron form factor for a two-component ($a = e, i$) plasma:

$$(\delta n_e \delta n_e)_{\omega,\mathbf{k}} = \frac{2n_e k^2}{\omega_e k_D^2}\left|\frac{1 + \overbrace{\chi_i}(\omega, \mathbf{k})}{\overbrace{\varepsilon}(\omega, \mathbf{k})}\right|^2 \mathrm{Im}\overbrace{\chi_e}(\omega, \mathbf{k}) + \left|\frac{\overbrace{\chi_e}(\omega, \mathbf{k})}{\overbrace{\varepsilon}(\omega, \mathbf{k})}\right|^2 \frac{\Theta_i}{\Theta_e}\frac{2n_e k^2}{\omega_i k_D^2}\mathrm{Im}\overbrace{\chi_i}(\omega, \mathbf{k}), \tag{17}$$

where we used for local equilibrium the following expression for the "source" $(\delta n_a \delta n_b)^S_{\omega,\mathbf{k}} = \delta_{ab}\frac{\Theta_a}{\omega_a}\frac{k^2}{2\pi e_a^2} \times \mathrm{Im}\overbrace{\chi_a}(\omega, \mathbf{k})$, and $\overbrace{\varepsilon}(\omega, \mathbf{k}) = 1 + \sum_a \overbrace{\chi_a}(\omega, \mathbf{k})$: $\overbrace{\chi_a}(\omega, \mathbf{k}) = (1 + i\frac{\partial}{\partial\omega}\frac{\partial}{\partial t} - i\frac{1}{k^2}\frac{\partial}{\partial r_i}k_j\frac{\partial}{\partial k_i}k_j)\chi_a(\omega, \mathbf{k}, t, \mathbf{r})$.

As above we can expand $\overbrace{\varepsilon}(\omega, \mathbf{k})$ near the plasma

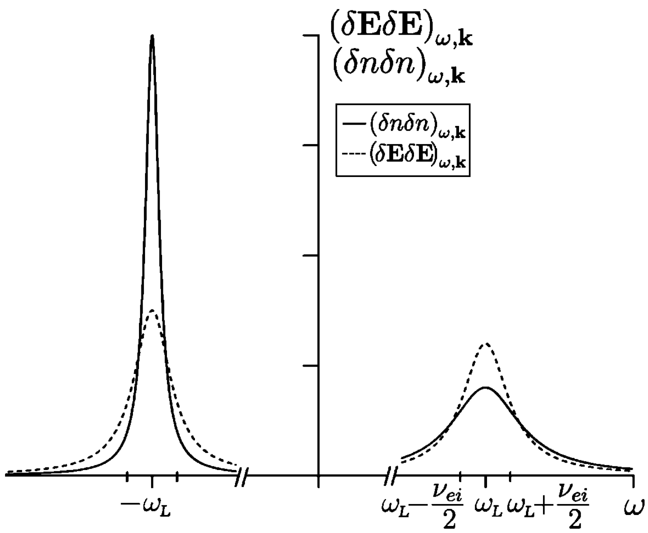

FIG. 1. The electron form factor $(\delta n_e \delta n_e)_{\omega,\mathbf{k}}$ (solid line) and the spectral function of electrostatic field fluctuations $(\delta\mathbf{E}\delta\mathbf{E})_{\omega,\mathbf{k}}$ (dashed line) as a function of frequency. $\mathbf{k}\cdot\frac{\partial n}{\partial \mathbf{r}} = \frac{\nu_{ei} n k_D^2}{54\omega_L}$; $\frac{k_D}{k} = 6$.

resonance $\omega = \omega_L$. Thus, for the electron line, $(\delta n_e \times \delta n_e)_{\omega,\mathbf{k}} = \frac{\overbrace{\gamma}}{(\omega - \mathrm{sgn}\,\omega_L)^2 + (\overbrace{\gamma})^2} \frac{2 n_e k^2}{\omega k_D^2 \partial\,\mathrm{Re}\varepsilon/\partial\omega}\Big|_{\omega=\omega_L}$, where

$$\overbrace{\gamma} = \left[\mathrm{Im}\varepsilon + \frac{\partial^2\,\mathrm{Re}\varepsilon}{\partial t\,\partial\omega} - \frac{1}{k^2}\frac{\partial}{\partial r_i}k_j \times \frac{\partial}{\partial k_i}k_j\,\mathrm{Re}\varepsilon\right] \Big/ \frac{\partial\,\mathrm{Re}\varepsilon}{\partial\omega}\Big|_{\omega=\omega_L\,\mathrm{sgn}\,\omega} \tag{18}$$

is the effective damping decrement for the electron form factor. At this stage of calculation, let us note that the damping decrements for the electrostatic field fluctuations [Eq. (14)] and for the electron density fluctuations [Eq. (18)] are not the same. The origin of this difference is that the Green function for electrostatic field fluctuation and density particle fluctuations are not the same. This property holds only in the inhomogeneous situation. An estimation for the plasma mode is then

$$\overbrace{\gamma} = \left[\nu_{ei} + \frac{2}{n}\frac{\partial n}{\partial t} + \frac{\omega_L}{nk^2}\mathbf{k} \cdot \frac{\partial n}{\partial \mathbf{r}}\left(1 + \frac{6k^2}{k_D^2}\right)\mathrm{sgn}\omega\right] \Big/ 2. \tag{19}$$

From this equation we see that the inhomogeneous correction in Eq. (19) is greater than the one in Eq. (15) by the factor $1 + k_D^2/6k^2$. For the same inhomogeneity, i.e., the same gradient of the density, we plot the form factor $(\delta n_e \delta n_e)_{\omega,\mathbf{k}}$ together with the $(\delta\mathbf{E}\delta\mathbf{E})_{\omega,\mathbf{k}}$ as functions of frequency (Fig. 1). This figure shows that the asymmetry of the spectral lines is present both for $(\delta n_e \delta n_e)_{\omega,\mathbf{k}}$ and for $(\delta\mathbf{E}\delta\mathbf{E})_{\omega,\mathbf{k}}$. However, this effect is more pronounced in $(\delta n_e \delta n_e)_{\omega,\mathbf{k}}$ than in $(\delta\mathbf{E}\delta\mathbf{E})_{\omega,\mathbf{k}}$. We have shown that the amplitude and width of the spectral lines of the electrostatic field fluctuations and form factor are affected by new nonlocal dispersive terms. They are not related to Joule dissipation and appear because of an additional phase shift between the vectors of induction and the electric field. This phase shift results from the finite time needed to set the polarization in the plasma with dispersion [9]. Such a phase shift in the plasma with space dispersion appears due to the medium inhomogeneity. These results are important for the understanding and the classification of the various phenomena that may be observed in applications; in particular, the asymmetry of lines can be used as a diagnostic tool to measure local gradients in the plasma.

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