Theoretical model of DC electric field formation in the ionosphere stimulated by seismic activity

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Abstract

Seismic activity is accompanied by emanation of soil gases into the atmosphere. These gases transfer positive and negative charged aerosols. Atmospheric convection of charged aerosols forms external electric current, which works as a source of perturbation in the atmosphere–ionosphere electric circuit. It is shown that DC electric field generated in the ionosphere by this current reaches up to 10 mV/m, while the long-term vertical electric field disturbances near the Earth's surface do not exceed 100 V/m. Such a limitation of the near-ground field is caused by the formation of potential barrier for charged particles at the Earth's surface in a process of their transport from soil to atmosphere. This paper presents the method for calculation of the electric field in the atmosphere and the ionosphere generated by given distribution of external electric current in the atmosphere.

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1. Introduction

Recent experimental and theoretical studies of pre-earthquake processes in the lithosphere, atmosphere and the ionosphere of the Earth display an existence of wide class of electromagnetic (EM) signals generated at different stages of earthquake development (Buchachenko et al., 1996). These so-called EM precursory signals include ULF magnetic field variations, seismic electric signals, anomalies in subionospheric VLF/LF propagation, bursts of pulse-like VLF emissions, ELF radiation correlated with plasma density irregularities in the ionosphere, disturbances of DC electric field both in the ionosphere and the atmosphere, etc. (see the most recent review by Varotsos (2001) and references therein). The present paper concerns only one of these phenomena—DC electric field in the atmosphere–ionosphere coupled system. Sorokin et al. (2001a) and Sorokin and Chmyrev (2002) have formulated the electrodynamic model of ionospheric precursors to earthquakes. This model gives an explanation to some electromagnetic and plasma phenomena connected with earthquake by the amplification of DC electric field in the ionosphere over a seismic region. To initiate these phenomena the electric field should reach up to 10 mV/m. Such electric fields were reported from satellite observations both over earthquake and hurricane regions (Chmyrev et al., 1989; Isaev et al., 2002). Possible connection of atmospheric electric field with seismic activity and the mechanisms of penetration of atmospheric field into the ionosphere...
were studied by Pierce (1976), Pulinets et al. (1994), Molchanov and Hayakawa (1996) and Boyarchuk et al. (1998). The calculation made in these papers show that the electric field in the ionosphere can reach 0.1–1.0 mV/m if vertical component of the field at the Earth surface exceeds 1–10 kV/m at the horizontal scale from 10 to 100 km. The ground-based observations have shown an existence of short-term (<1 h) seismic-related vertical electric field disturbances with magnitudes up to 1000 V/m, while the long-term (1–10 days) electric field perturbations exceeding ~100 V/m within earthquake areas at distances from ten to few hundreds km from the epicenter have not been observed (Vershinin et al., 1999). This contradiction between satellite and ground-based measurements of long-term vertical DC electric field disturbances can be reconciled by the consideration of the effects of external electric current in the atmosphere, as will be carried out in present paper.

Sorokin and Yaschenko (2000) and Sorokin et al. (2001a, b) have constructed the theoretical model of the electric field disturbances caused by the conductivity currents in the atmosphere and the ionosphere initiated by external electric current. According to this model, external current arises as a result of emanation of charged aerosols transported into the atmosphere by soil gases and subsequent processes of upward transfer, gravitational sedimentation and charge relaxation. So the time scales of variations of the external currents and the injection of soil gases should be similar. Seismic-induced emanation enhancement of soil gases and accompany elements is observed during days or week before earthquake (Alekseev and Alekseeva, 1992; Pulinets et al., 1994; Virk and Singh, 1994; Heincke et al., 1995). The model estimate of ionospheric electric field caused by pre-earthquake processes give the magnitude ~10 mV/m (Sorokin et al., 2001a). Further development of this model includes a new method for calculation of the electric field in the atmosphere and the ionosphere and the mechanism for limitation of vertical electric field near the Earth surface is given below.

2. Equation for the electric field potential

Let us consider generation of the electric field by external current \( j_e \) in the Earth–ionosphere layer. We will derive the equations for potential \( \phi \) of the electric field disturbance \( E = -\nabla \phi \). Let us introduce the Cartesian co-ordinates \((x, y, z)\) with the axis \( z \) directed vertically upward parallel to homogeneous magnetic field and with the origin located on the absolutely conductive Earth’s surface (see Fig. 1). We assume that the electric field potential on this surface is \( \phi|_{z=0} = 0 \). The atmosphere characterized by the conductivity \( \sigma(z) \) is located in the layer \( 0 < z < z_1 \). Electric potential in this region is derived from the equation of continuity and the

\[
\nabla \cdot \mathbf{j} = 0, \quad j = \sigma E = -\sigma \nabla \phi
\]

and satisfies the following equation:

\[
\frac{\partial^2 \phi}{\partial z^2} + \frac{1}{\sigma(z)} \frac{\partial \sigma}{\partial z} \frac{\partial \phi}{\partial z} + \Delta \phi = \frac{1}{\sigma(z)} \nabla \cdot \mathbf{j}.
\]

(1)

Plane \( z = z_1 \) coincides with thin ionosphere characterized by the tensor of integral conductivity with the components \( \Sigma_p \) and \( \Sigma_H \)—Pedersen and Hall conductivity, respectively. Integrating the equation of current continuity in the ionosphere \( \nabla \cdot \mathbf{j} = 0 \) on its width gives the boundary condition:

\[
j_z(z_1 + 0) - j_z(z_1 - 0) = \Sigma_p \Delta \phi_1,
\]

(2)

where \( \phi_1 = \phi(z_1) \). Since in quasi-static approximation the geomagnetic field lines in the magnetosphere are equipotential in the distribution of the electric potential in the ionosphere and the magnetic field-aligned current on its upper boundary are transported into magnetically conjugate region without changes. Field-aligned electric current flowing in the magnetosphere is closed by transverse conductivity currents in the conjugate ionosphere and the atmosphere. The boundary condition on the conjugate ionosphere is similar to Eq. (2), while the equation for potential in the conjugate atmosphere coincides with (1) at \( j_e = 0 \). Let the altitude dependence of atmospheric conductivity has the form:

\[
\sigma(z) = \sigma_0 \exp(z/h),
\]

(3)

the external current is directed upward and has symmetric distribution relative to the \( z \)-axis: \( j_e = j_e(z, r) \), where \( r = \sqrt{x^2 + y^2} \). Applying Gankel
transform of zero order
\[ \Phi(k, z) = \int_{0}^{\infty} \phi(r, z)J_{0}(kr)r\, dr \]
to Eq. (1) and the boundary condition (2) we obtain
\[ \frac{d^{2}\Phi}{dz^{2}} + \frac{1}{h} \frac{d\Phi}{dz} - k^{2}\Phi = \frac{1}{\sigma} \frac{dj_{m}}{dz}, \quad 0 < z < z_{1}, \]
\[ \Phi|_{z=0} = 0, \quad j_{m} + \sigma_{1}\frac{d\Phi}{dz}|_{z=z_{1}} = -\Sigma k^{2}\Phi_{1}, \quad (4) \]

where \( j_{m} \) is field-aligned electric current in the magnetosphere. Let \( z' \) is upward directed axis of Cartesian coordinates in the conjugate atmosphere. The equations for potential and the boundary conditions in this region have the form:
\[ \frac{d^{2}\Phi}{dz'^{2}} + \frac{1}{h} \frac{d\Phi}{dz'} - k^{2}\Phi = 0, \quad 0 < z' < z_{1}, \]
\[ \Phi|_{z'=0} = 0, \quad -j_{m} + \sigma_{1}\frac{d\Phi}{dz'}|_{z'=z_{1}} = -\Sigma k^{2}\Phi_{1}. \quad (5) \]

Solution of equation for potential (5) in the conjugate atmosphere satisfying the boundary condition on the Earth’s surface is as follows:
\[ \Phi = \Phi_{1}\exp\left[-\frac{z' - z_{1}}{2h}\right]\sinh(qz'), \quad q = \sqrt{k^{2} + \frac{1}{4h^{2}}}. \]
Using this solution in boundary condition (5) yields the field-aligned current in the magnetosphere:
\[ j_{m} = \Phi_{1}[\Sigma k^{2} + \sigma_{1}(q\coth(qz_{1}) - 1/2h)], \quad \sigma_{1} = \sigma(z_{1}). \]

Substitution of \( j_{m} \) in boundary condition (4) yields boundary condition for the electric field potential on the lower edge of the ionosphere. Let us derive the equation and boundary conditions for calculations of the electric field distribution in the Earth–ionosphere layer initiated by external current with consideration of conjugate atmosphere:
\[ \frac{d^{2}\Phi}{dz^{2}} + \frac{1}{h} \frac{d\Phi}{dz} - k^{2}\Phi = \frac{1}{\sigma} \frac{dj_{m}}{dz}, \]
\[ \phi(r, z) = \int_{0}^{\infty} \Phi(k, z)J_{0}(kr)k\, dk, \]
\[ \Phi|_{z=0} = 0, \quad \left[ \frac{d\Phi}{dz} + a(k)\Phi \right]_{z=z_{1}} = 0, \]
\[ a(k) = \frac{2\Sigma k^{2} + q\coth(qz_{1}) - 1}{2h}. \quad (6) \]

Influence of conjugate region to the field distribution in the atmosphere is considered through boundary condition (6). These equations will be used below for checking the accuracy of approximate calculations.

3. Approximate method for the field calculation

Formation of external electric current in the atmosphere over seismic region is connected with turbulent upward transport of charged aerosols injected from soil, their gravitational sedimentation and neutralization of charges. Growth of seismic activity intensifies aerosol injection with the soil gases. This intensification process spreads over an area from 10 to 100 km in diameter. For such processes Eq. (1) with the boundary condition (2) can be solved approximately for any dependence of conductivity on altitude. Taking into account that the last term in the left-hand side of Eq. (1) is of the order of \( \sim\Phi/r_{0}^{2} \) and two other terms are \( \sim\Phi/h^{2} \) (where \( r_{0} \) is characteristic horizontal scale of the disturbed area and \( h \) is a scale of altitude variation of the atmosphere conductivity), we can neglect the last term in comparison with other two. As result we obtain

\[ \frac{d}{dz}\left[ \sigma(z) \frac{d\phi}{dz} - j_{c}(z, r_{1}) \right] = 0. \quad (7) \]

Let us find the boundary conditions for potential on the plane \( z = z_{1} \). Condition of current continuity (2) in conjugate regions of the ionosphere has the form
\[ j_{m} + \sigma_{1}\frac{d\phi}{dz}|_{z=z_{1}} = \Sigma p\Delta_{\perp}\varphi_{1}, \]
\[ -j_{m} + \sigma_{1}\frac{d\phi}{dz}|_{z'=z_{1}} = \Sigma p\Delta_{\perp}\varphi_{1}, \quad (8) \]

where \( \Delta_{\perp} \) is Laplace operator in the plane \( z = z_{1} \). Solution of Eq. (7) at the condition that \( j_{c} = 0 \) in conjugate region yields
\[ \sigma_{1}\frac{d\phi}{dz}|_{z=z_{1}} = \varphi_{1}/\rho, \quad \rho = \int_{0}^{z_{1}} \frac{dz}{\sigma(z)}. \quad (9) \]

Substituting Eq. (9) in Eq. (8) and adding Eq. (8) we find the boundary conditions on the ionosphere for Eq. (7):
\[ \varphi_{z=0} = 0, \quad \sigma_{1}\frac{d\phi}{dz}|_{z=z_{1}} = 2\Sigma p\Delta_{\perp}\varphi_{1} - \varphi_{1}/\rho, \quad (10) \]

where \( \varphi_{1} = \varphi(z_{1}, r_{1}) \) is horizontal distribution of potential in the ionosphere. Eq. (7) is similar to 1D equation obtained by Sorokin et al. (2001a). The only difference is in the form of boundary condition (10) on the lower ionospheric boundary \( (z = z_{1}) \), which takes into account the transverse spreading of current in the ionosphere at \( z \geq z_{1} \). This leads to the generation of horizontal component of the electric field in the ionosphere.
Solution of Eq. (7) satisfying the condition \( \varphi_{z=0} = 0 \) has the form

\[
\varphi(r_\perp, z) = \int_0^z \frac{j_1(r_\perp, z')}{\sigma(z')} \, dz' - j_1(r_\perp) \int_0^z \frac{dz'}{\sigma(z')}.
\]

\[j_1(r_\perp) = \frac{\varepsilon(r_\perp) - \varphi_1(r_\perp)}{\rho}, \quad \varepsilon(r_\perp) = \int_0^z j_1(r_\perp, z') \, dz', \tag{11}\]

where \( j_1(r_\perp) \) is an electric current on the lower edge of the ionosphere. This current flows into the ionosphere from the atmosphere. Using Eq. (11) and the boundary condition (10) yields the equation for horizontal distribution of the ionosphere potential \( \varphi_1 \):

\[
\Delta_{\perp} \varphi_1 - \frac{1}{2\Sigma_p \rho} \varphi_1 = - \frac{j_1(r_\perp)}{2\Sigma_p}. \tag{12}\]

Estimates show that the second addendum in the left-hand side of the obtained equation is negligibly small when \( r_0 \ll 10^8 \) m that practically means always. Therefore the equation for the ionosphere potential distribution in the co-ordinate presentation has the form of Poisson equation:

\[
\Delta_{\perp} \varphi_1 = - \frac{j_1}{2\Sigma_p}. \tag{12}\]

Eqs. (11) and (12) are applicable for calculations of the electric fields connected with external current of arbitrary distribution in horizontal plane and for any altitude distribution of atmospheric conductivity, when the horizontal spatial scale of current exceeds the altitude of lower edge of the ionosphere.

4. Amplification of the electric field in the ionosphere at its stable magnitude on the Earth surface

Let us consider large-scale external electric current with axial-symmetric distribution in horizontal plane. Eq. (12) yields the expression for radial component of the electric field in the ionosphere:

\[
E_r(r) = \frac{1}{2\Sigma_p r} \int_0^r \frac{r'}{j_1(r')} \, dr'.
\]

\[j_1(r) \approx \frac{\varepsilon}{\rho} = \frac{1}{\rho} \int_0^z j_1(r, z) \, dz, \tag{13}\]

which is determined by the conductivity current \( j_1(r) \) flowing into the ionosphere. The vertical electric field component in the Earth–ionosphere layer is derived from Eq. (11):

\[
E_z(r, z) = \frac{1}{\sigma(z)} \left[ j_1(r) - j_\varphi(r, z) \right] = \frac{1}{\sigma(z)} \left[ \frac{1}{\rho} \int_0^z j_1(r, z) \, dz - j_\varphi(r, z) \right]. \tag{14}\]

To check the accuracy of approximate method for the field calculations the comparison of different horizontal distributions of vertical electric field component \( E_z(r, z = z_1) \) at the altitude of lower ionospheric boundary \( z = z_1 \) was carried out. The graphs for functions derived from Eq. (14) and the function \( E_z(r, z = z_1) = -\partial \varphi(r, z)/\partial z \) were compared, where \( \varphi(r, z) \) was found from exact numerical solution of Eq. (6). In both cases it was assumed that

\[
j_\varphi(r, z) = j \exp(-z/\tilde{h} - r^2/\tilde{r}_0^2), \quad \sigma = \sigma_0 \exp(z/\delta),
\]

where \( \tilde{h} = 5 \) km and \( \tilde{r}_0 = 15 \) km. For calculations we assumed \( \Sigma_p = 2 \times 10^{12} \) cm/s, \( d = 2\Sigma_p/\sigma_1 = 1.6 \times 10^{10} \) cm. Comparison was made for three cases corresponding to \( r_0 = 50, 100 \) and 150 km. It appeared that the relative errors of approximate method for these three cases were correspondingly, 20, 7 and 3%. Thus, the approximate method for calculations of the electric field can be used with sufficient accuracy if horizontal scale of external current exceeds 100 km.

As mentioned above, the simultaneous observations of the electric field variations at two ground-based stations separated by 10–100 km have shown that the magnitude of vertical electric field variation with characteristic time scale of 1 day and more did not exceed 100 V/m (Vershinin et al., 1999). We will assume that the seismic-related horizontal electric field in the ionosphere can reach 10 mV/m (Chmyrev et al., 1989). Let us show that such a relationship between the fields in the ionosphere and on the ground can be provided by a sum of external currents arising from the injection of positive and negative charged aerosols into the atmosphere:

\[
j_\varphi(r, z) = j_p(r)s_p(z) - j_n(r)s_n(z), \quad s_p(z = 0) = s_n(z = 0) = 1. \tag{15}\]

Functions \( s_p(z), s_n(z) \) denote the altitude distributions of external currents. Substitution of Eq. (15) in Eq. (14) yields

\[
j_1(r) = \frac{1}{\rho} \left[ j_p(r)k_p - j_n(r)k_n \right],
\]

\[
E_{zo}(r) = \frac{1}{\sigma(0)} \left[ j_1(r) - j_p(r) + j_n(r) \right],
\]

\[
k_{pn} = \int_0^z \frac{d}{dz} \frac{s_{p,n}(z)}{\sigma(z)}, \quad E_{zo}(r) = E_z(r, z = 0). \tag{16}\]

Expressions (16) present a set of linear equations for determination of \( j_p, j_n \) at the given values of \( j_1, E_{zo} \).
corresponding to observational data. Let us consider the bound case. Let \( E_{\infty} = 0 \) and the horizontal electric field in the ionosphere is 10 mV/m. According to Eq. (13) this value corresponds to definite magnitude of conductivity current \( j_s \) flowing into the ionosphere. Assuming the exponential dependence of atmospheric conductivity (3) on altitude and \( E_{\infty} = 0 \) in Eq. (16) we find

\[
j_p = j_1 \frac{\rho - k_n}{k_p - k_n}, \quad j_n = j_1 \frac{\rho - k_p}{k_p - k_n}.
\]  

(17)

Substitution of Eq. (17) in Eq. (14) yields the height distribution of vertical component of the electric field in the Earth–ionosphere layer for the case of \( E_{\infty} = 0 \):

\[
E_r(z) = \frac{j_1(r)}{\sigma(z)} \left[ 1 - \frac{\rho - k_n}{k_p - k_n} s_p(z) + \frac{\rho - k_p}{k_p - k_n} s_n(z) \right].
\]  

(18)

Radial distribution of horizontal component of the ionospheric electric field is determined by Eq. (13). Using Eq. (16) in Eq. (13) we find

\[
E_i(r) = \frac{1}{2\Sigma_p \rho f_0} \int_0^r \frac{d\tau'}{r' \left[ k_p j_p(r') - k_n j_n(r') \right]}. 
\]  

(19)

It is seen from Eq. (19) that horizontal electric field in the ionosphere is determined by external currents in the near-Earth atmosphere. Let us assume that the height dependence of external currents and atmospheric conductivity have the form:

\[
s_{p,n} = \exp(-z/h_{p,n}), \quad \sigma(z) = \sigma_0 \exp(z/h), 
\]  

(20)

where \( h_{p,n} \) is a vertical scale of spatial distribution of external currents in the atmosphere. From Eq. (16) we obtain

\[
\rho = \frac{h}{\sigma_0}, \quad k_p = \frac{h h_0}{\sigma_0 (h_0 + h)}, \quad k_n = \frac{h h_0}{\sigma_0 (h_0 + h)}. 
\]  

(21)

Let us select the horizontal distribution of external currents on the Earth surface in the form:

\[
j_p(r) = j_{p0} \exp(-r^2/r_0^2), \quad j_n(r) = j_{n0} \exp(-r^2/r_0^2). 
\]

These currents generate the flow into the ionosphere current and the electric field. Their horizontal structure is described by the following expressions:

\[
E_i(r) = E_{i0} \frac{r_0}{r} \left[ 1 - \exp \left( -\frac{r^2}{r_0^2} \right) \right], 
\]  

\[
E_{i0} = \frac{r_0}{4 \Sigma_p} \frac{h_p}{h + h_0} j_{p0} - \frac{h_n}{h + h_0} j_{n0}, 
\]  

\[
j_i(r) = j_{i0} \exp \left( -\frac{r^2}{r_0^2} \right), 
\]  

\[
j_{i0} = \frac{h_p}{h + h_0} j_{p0} - \frac{h_n}{h + h_0} j_{n0}. 
\]  

(22)

According to Sorokin et al. (2001a) magnitude of external current on the Earth’s surface is determined by the formulas:

\[
\begin{align*}
  j_{p0} &= 4\pi \sigma_0 e \frac{Z_p h_p}{h_0}, \\
  j_{n0} &= 4\pi \sigma_0 e \frac{Z_n h_n}{h_0}. 
\end{align*}
\]

where \( e Z_{p,n} \) is a particle electric charge and \( N_{p,n0} \) is a number density of particles near the ground. \( E_{i0} \) can be presented in the following form:

\[
E_{i0} = \frac{\pi r_0 \sigma_0 e Z_p N_{p0}}{\Sigma_p} \frac{h_p^2}{h + h_p} (1 - A), 
\]  

\[
A = \frac{\rho_0 (h + h_0) j_{p0}}{h_p (h + h_0) j_{p0}}. 
\]  

(23)

The estimate of a magnitude of the electric field in the ionosphere will be made at the following parameters: \( h_p = 20 \text{ km}, \quad h_n = 15 \text{ km}, \quad h = 5 \text{ km}, \quad r_0 = 100 \text{ km}, \quad N_{p0} = 4 \times 10^3 \text{ cm}^{-3}, \quad Z_p = 100, \quad \sigma_0 = 2 \times 10^{-4} \text{ c}^{-1}, \quad \Sigma_p = 2 \times 10^{13} \text{ cm/s}, \quad j_{p0}/j_{n0} = 0.5. \) Substitution of these values in Eq. (23) gives \( E_{i0} = 15 \text{ mV/m} \). Fig. 2 presents the spatial distributions of the horizontal electric field in the ionosphere and the external current on the Earth surface derived from formulas (22). As it is seen from Fig. 2 the ionospheric electric field can reach \( \sim 10 \text{ mV/m} \). The calculations show that it is possible to select such a set of the parameters of external current, which provides the amplification of horizontal electric field in the ionosphere up to 10 mV/m and leaves the vertical electric field on the Earth surface unchanged.

5. Mechanisms of limitation of the electric field on the Earth surface

As it is noted above the significant (up to 1 kV/m) pre-earthquake vertical electric fields on the Earth surface have characteristic temporal scale less or of the order of 1 h (Vershinin et al., 1999). At the same time the atmospheric electric field variations with typical scale exceeding 1 day at the distances within 10–100 km from earthquake center during seismically active period are characterized by the magnitudes not exceeding \( \sim 100 \text{ V/m} \) (Vershinin et al., 1999). The cause of such limitation can be explained by the mechanism of feedback between disturbances of vertical electric field and the causal external currents on the Earth surface. Such a feedback is caused by the formation of potential barrier on the ground-atmosphere boundary at the passage of upward moving charged aerosols through this boundary. Their movement upward is performed due to viscosity of soil gases flowing into the atmosphere. If for example positively charged particle goes from ground to the atmosphere, the Earth surface is charged negatively. Downward electric field that prevents more particle penetration through the surface (see Fig. 3 for illustration). At the same time this field stimulates the going out on the surface of the negatively charged particles. In a presence of such coupling the magnitudes of external currents (15) on the Earth surface depend on vertical component of the electric field on the
surface:

\[ j_p(r, E_{z0}(r)) = j_{p0}(r)f(E_{z0}(r)/E_{cp}), \]
\[ j_n(r, E_{z0}(r)) = j_{n0}(r)f(-E_{z0}(r)/E_{cn}), \] (24)

where \( j_{p0} \) and \( j_{n0} \) are determined by the injection intensity of aerosols in missing of the electric field influence. Qualitatively the function \( f(E_{z0}/E_c) \) characterizing the electric field effect could be presented in a form shown in Fig. 4. The graphs show that when negative electric field reaches the critical value \( E_{cp} \) it stops the current of positive particles, while positive electric field stops negative particles current. Critical field can be estimated from the balance between viscosity, gravity and electrostatic forces. Viscosity force connected with elevated soil gases acts in upward direction. Gravity force is directed downward. Electrostatic force connected with going out of positive particle is directed downward,

\[ E_{cp} = (6\pi \eta R_p V - m_p g)/eZ_p, \]
\[ E_{cn} = (6\pi \eta R_n V - m_n g)/eZ_n, \] (25)

where \( \eta \) is air viscosity coefficient, \( V \) is the velocity of elevation of soil gases within ground, \( R_p, n \) is radii of aerosol particles, \( m_{p,n} = (4/3)\pi R_{p,n}^3\mu \) is particle masses and \( \mu \) is their number density. For simplicity we will assume that the aerosols of opposite signs consist from
the same particles \((E_{cp} = E_{cn} = E_c)\). For further calculations we select \(f(E_{z0}/E_c)\) in the form \(f = \sqrt{1 + E_{z0}/E_c}\) given in Fig. 4. Substitution of this function in Eqs. (24) and (16) yields:

\[
E_{z0}(r) = \frac{1}{\sigma_0} \left[ j_{z0}(r) \left( \frac{k_p}{\rho} - 1 \right) \sqrt{1 + \frac{E_{z0}(r)}{E_c}} - j_n(r) \left( \frac{k_n}{\rho} - 1 \right) \sqrt{1 - \frac{E_{z0}(r)}{E_c}} \right].
\]

This equation allows one to calculate the vertical electric field component on the Earth surface at the given values of \(j_{z0}, j_{n0}\). Let us rewrite Eq. (26) in the form

\[
E_{z0} = -\frac{j_{z0}}{j_{n0}} \frac{\rho - k_p}{\rho - k_p} \frac{\sigma_0}{\sigma_n} \left( \sqrt{1 + \frac{E_{z0}}{E_c}} - B \sqrt{1 - \frac{E_{z0}}{E_c}} \right),
\]

\[
B = \frac{\rho - k_n}{\rho - k_p} j_{n0}.
\]

For estimates of the field magnitudes we will use the height distributions (20) and the formulas (21). Fig. 5 presents the dependence of vertical electric field derived from Eq. (27) on dimensionless current \(J\) on the Earth surface:

\[
J = j_{z0}/(\rho - k_p)/\sigma_0 E_c \\
= j_{p0}/\rho_0 E_c (h + h_p)/\rho_0 = 0.8 j_{p0}/\sigma_0 E_c.
\]

It is seen from this plot that the field magnitude at any currents does not exceed the maximum value:

\[
E_{zm} = -E_c(1 - B^2)/(1 + B^2).
\]

For the parameters \(j_{z0}/j_{p0} = 0.64\) and \(B = (h + h_p)/j_{p0}/(h + h_n)j_{n0}\) we find \(E_{zm} = -0.2 E_c\). Let us estimate the critical field (25). Assuming \(\eta = 1.72 \times 10^{-4} \text{g/cm s, } V = 0.01 \text{cm/s, } R = 5 \times 10^{-5} \text{ cm, } \mu = 1.5 \text{g/cm}^3, \text{ } Z = 100\) we obtain \(E_c = 0.015 \text{eV/cm, } \sigma_0 E_c = 10 \text{pA/cm}^2\). Therefore the vertical electric field on the Earth surface cannot exceed \(E_{zm} = 90 \text{V/m.}\) Horizontal component of the electric field in the ionosphere at the given currents \(j_{z0}, j_{n0}\) can be found by substitution of Eq. (24) in Eq. (19):

\[
E_r(r) = \frac{1}{2 \Sigma_p \rho_p} \int_0^r dr' \left[ k_p j_{p0}(r') \sqrt{1 + \frac{E_{z0}(r')}{E_c}} - k_n j_{n0}(r') \sqrt{1 - \frac{E_{z0}(r')}{E_c}} \right].
\]
Substitution of Eqs. (15) and (16) in Eq. (14) and use of Eq. (24) yields the vertical electric field in the Earth–ionosphere layer:

\[
E_z(r, z) = \frac{1}{\sigma(z)} \left[ \left( \frac{k_p}{\rho} - s_p(z) \right) j_{p0}(r) \sqrt{1 + \frac{E_{zo}(r)}{E_c}} - \left( \frac{k_n}{\rho} - s_n(z) \right) j_{n0}(r) \sqrt{1 - \frac{E_{zo}(r)}{E_c}} \right].
\] (30)

The surface electric field \(E_{zo}(r)\) in Eqs. (29) and (30) is determined by Eq. (26). If the external current parameters satisfy the condition \(B = 1\) then Eq. (27) yields \(E_{zm} = 0\), that is vertical component of the electric field on the Earth surface is zero. The above condition coincides with Eq. (17) and Eq. (30) transforms to Eq. (18). This particular event was considered above.

Distributions of the vertical electric field on the Earth’s surface and the horizontal electric field in the ionosphere calculated from Eqs. (29) and (30) are presented in Fig. 6. Dependence of external currents formed by positive and negative particles on radial distance \(r\) was assumed as

\[
j_p(r) = j_{p0} \exp(-r^2/r_0^2), \quad j_n(r) = j_{n0} \exp(-r^2/r_0^2),
\]

\[
j_{p0} = 4\pi\sigma_0 e Z_p h_p N_{p0}, \quad j_{n0}/j_{p0} = 0.64.
\]

The altitude distributions of external currents and atmospheric conductivity are determined by Eq. (20), the parameters \(\rho, k_p, k_n\) are given in Eq. (21). For numerical estimates we select the following parameters: \(h_p = 20\) km, \(h_n = 15\) km, \(h = 5\) km, \(r_0 = 100\) km, \(N_{p0} = 8 \times 10^3 \text{ cm}^{-3}\), \(Z_p = 100\), \(\sigma_0 = 2 \times 10^{-8} \text{ s}^{-1}\), \(\Sigma_p = 2 \times 10^3 \text{ cm/s}\), \(E_c = 0.015 \text{ cgs} = 450 \text{ V/m}\). It is seen from Fig. 6 that the horizontal electric field in the ionosphere reaches \(\sim 10 \text{ mV/m}\), while the vertical electric field on the Earth surface is limited by magnitude \(\sim 100 \text{ V/m}\). The calculations show that the ionospheric field reaches maximal magnitudes at the edges of area of external current. The horizontal scale of vertical electric field enhancement on the ground exceeds about three times the characteristic horizontal scale of external current. Within this area the vertical field practically does not depend on distance.

6. Conclusions

According to theoretical model (Sorokin et al., 2001a; Sorokin and Chmyrev, 2002) the electromagnetic and plasma precursors to earthquakes observed within days to weeks before earthquakes are caused by the enhancement of DC electric field in the ionosphere up to \(10 \text{ mV/m}\) (see also Sorokin et al., 1998, 2000; Sorokin and Chmyrev, 1999; Borisov et al., 2001). Such electric fields were reported from the satellite observations both over earthquake and hurricane regions (Chmyrev et al.,
1989; Isaev et al., 2002). The ground-based observations did not reveal any significant long-term (1–10 days) electric field disturbances within earthquake area at the distances of 10–100 km from epicenter (Vershchin et al., 1999). This contradiction between satellite and ground electric field observations is avoided by consideration of the effects of external electric current in the atmosphere made in the present paper.

It was shown that this current works as a source of the electric field disturbances both in the ionosphere and on the ground. The external current is excited in a process of vertical atmospheric convection of charged aerosols injected with elevated gases from soil into the atmosphere. Its inclusion into the atmosphere–ionosphere electric circuit leads to such redistribution of conductivity current that resulted electric field on the Earth’s surface spreads over the region exceeding the horizontal scale of external current (or zone of aerosols injection) about three times. This means that the disturbed electric field can be observed outside the active area where injection of aerosols and their vertical transport took place. The field magnitude within this area practically does not depend on distance.

Aerosols are injected into the atmosphere due to intensifying soil-gas elevation during the enhancement of seismic activity. The field limitation on the Earth’s surface is caused by feedback mechanism between excited electric field and the causal external current. This feedback is produced by the potential barrier for charged particle at its transfer from ground to the atmosphere.

The presented model is made in approximation of vertical magnetic field to simplify the calculations. Perhaps it makes direct comparison of the results with experimental data not quite correct because earthquakes occur mainly at mid and low latitudes. However, the assumption of vertical field does not influence significantly on the principle result showing the effects of external electric current on the magnitudes and distribution of DC electric field in the near Earth environment.

The effect of limitation of the vertical electric field magnitude on the ground creates significant advantage for satellite monitoring of seismic related electric field disturbances in the atmosphere as compare to ground-based observations. Besides, an amplification of pre-earthquake electric field in the ionosphere can be verified by simultaneous measurements of other electromagnetic and plasma effects sensible to growth of DC electric field. Thus the ionosphere can be more efficient indicator of definite class of earthquake precursors than the ground-based observations. Combination of satellite measurements of the ionospheric precursors with simultaneous ground-based observations of seismic electric signals, ULF magnetic oscillations, subionospheric VLF/LF signal disturbances and other phenomena could be a powerful tool for the development of earthquake monitoring and forecasting methods.

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References


