Законы масштабирования конвекции и магнетизма в недрах Земли и планет

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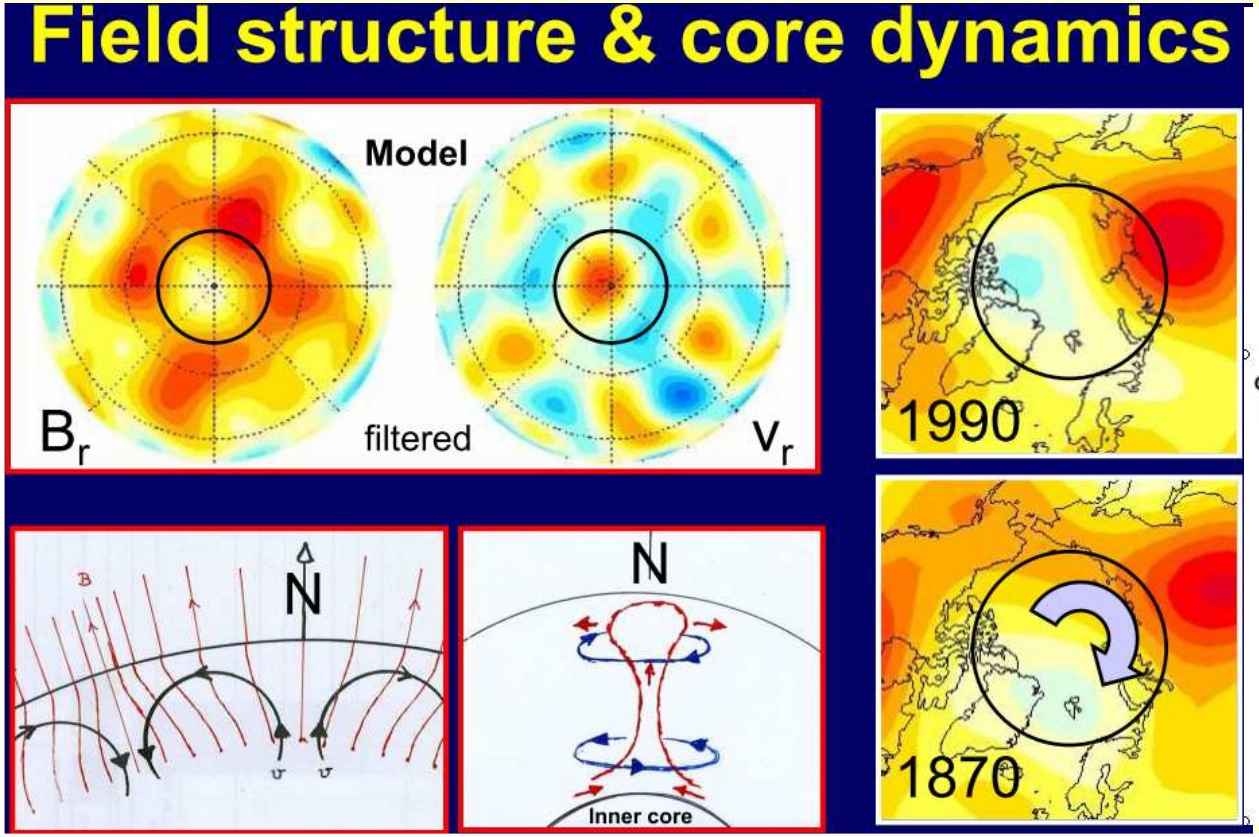
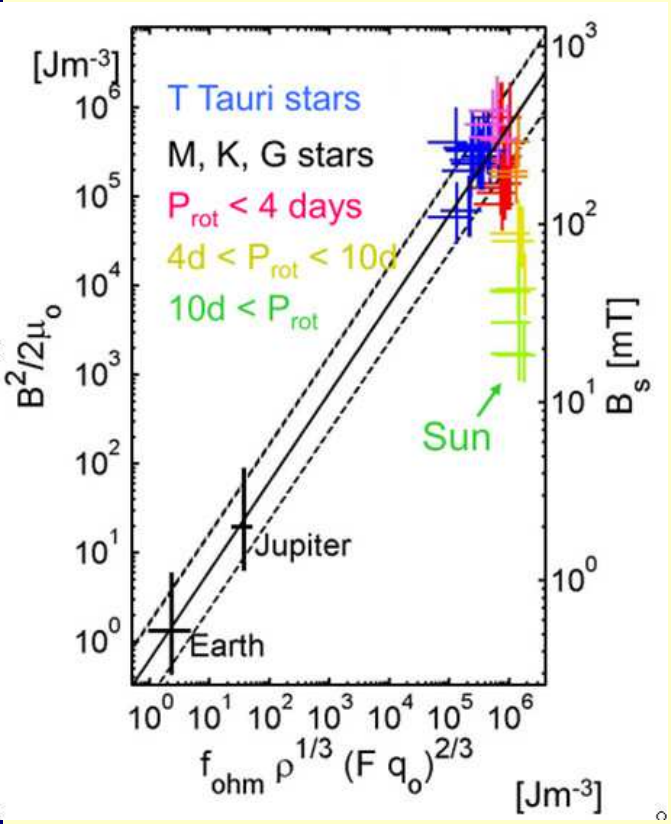
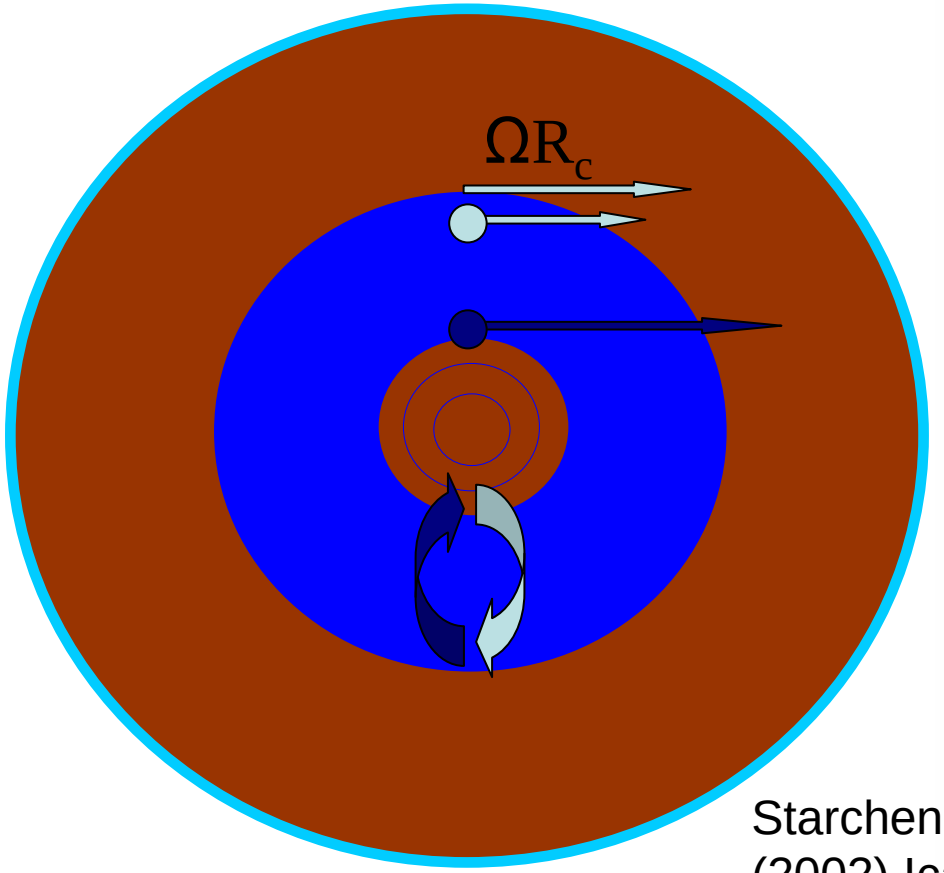
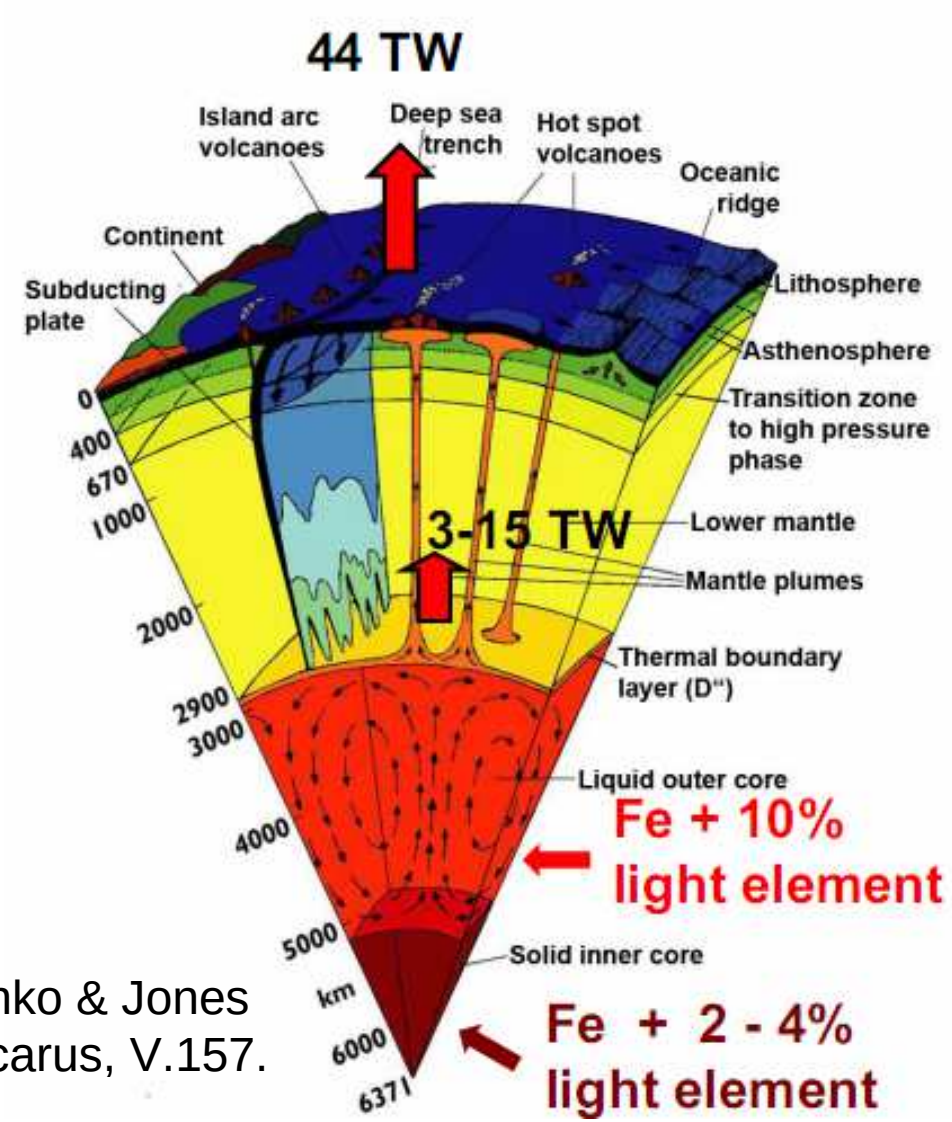




КОНВЕКЦИЯ тепловая композиционная

Брагинский С.И. (1963)





Starchenko & Jones (2002) Icarus, V.157.

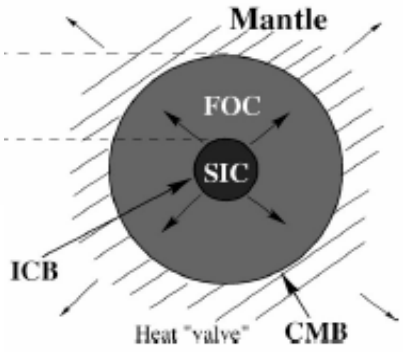
Скорость*Ускорение =Поток плавучести=Мощность/Масса $VA = F$ (1)

Adiabatic $(\overline{S}' = 0)$**, well-mixed** $(\overline{\xi}' = 0)$**, hydrostatic** $(\overline{p}' = \overline{\rho} g)$ **Reference State:**

$d\overline{T}/dr \equiv \overline{T}' = \alpha^S \overline{g}(r)$, $d\overline{\mu}/dr \equiv \overline{\mu}' = \alpha^\xi \overline{g}(r)$.

Earth: $\alpha^S \approx 7 \cdot 10^{-5} K s^2/m^2$, $\alpha^\xi \approx 0.6$;

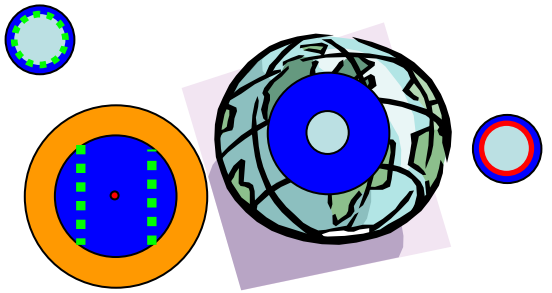
Jupiter/Saturn: $\alpha^S = \alpha \overline{T}/c_p \sim 10^{-5} K s^2/m^2$.



Very small viscosity & diffusivity:

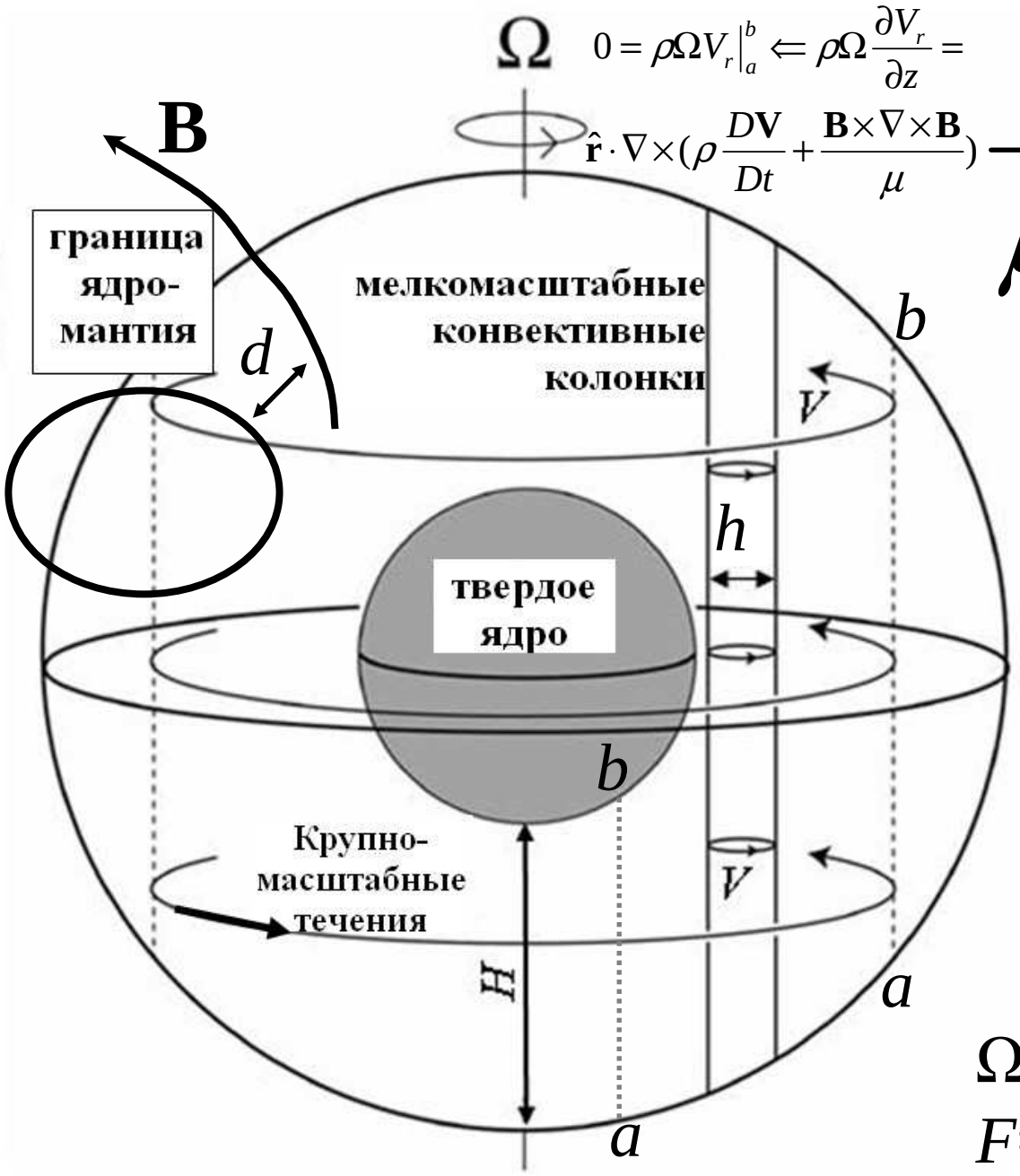
$\nu \sim 10^{-15} H^2 \Omega$

$\kappa \sim 10^{-9} HV$



Not so small magnetic diffusivity

$\eta = 1/\mu\sigma \sim (10^{-2} \ldots 10^{-3})\, HV$



$$0 = \rho\Omega V_r\big|_a^b \Leftarrow \rho\Omega\frac{\partial V_r}{\partial z} =$$

$$\hat{\mathbf{r}}\cdot\nabla\times(\rho\frac{D\mathbf{V}}{Dt}+\frac{\mathbf{B}\times\nabla\times\mathbf{B}}{\mu})$$

$$\frac{B^2}{\mu\rho d^2} = \frac{V^2}{h^2} \quad (2)$$

$$\frac{\Omega V}{H} = \frac{A}{h} \quad (3)$$

$$\frac{A}{h} = \frac{V^2}{h^2} \quad (4)$$

Ω = 73/Ms, H = 2.3 Mm, F=(1±0.8)pW/kg для жидкого ядра Земли

Inertia=Archimedean=Coriolis

(1) & (3-4)

SCALING LAWS from > 145 magnetic dynamo simulations [Olson and Christensen (2006) EPSL 250; Christensen and Aubert (2006) Geoph. J. Int. 166; Christesen *et al.* (2009) Nature 457]

$$A = VV / h = 0.5 \text{ nm/s}^2$$

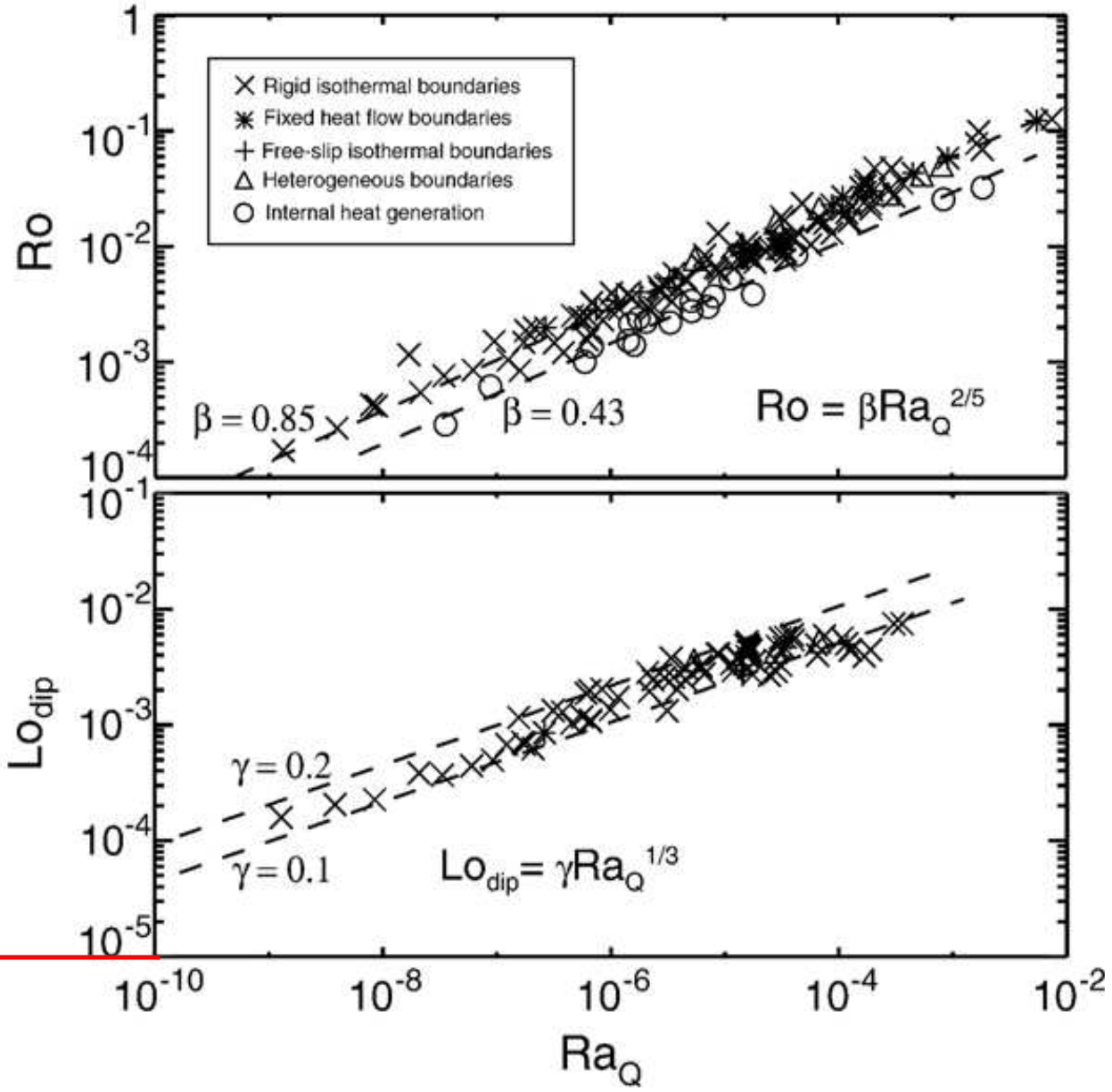
$$d = (FH^3 / \Omega^3)^{1/5} = 8 \text{ km}$$

$$V = (F^2 H / \Omega)^{1/5} = 2 \text{ mm/s}$$

$$\frac{(0.4\,\text{mT})^2}{\mu \rho h} = 2 \text{ nm/s}^2$$

Olson and Christensen (2006) EPSL 250

Fig. 2. Fluid velocity and dipole moment versus buoyancy flux Rayleigh number from numerical dynamos. Top: Rossby number for fluid velocity. Dashed lines show fits to 2/5-power-law for base-heated and internally heated cases, respectively. Bottom: Lorentz number for dipole moment from base-heated, dipole-dominant dynamos with Nusselt numbers > 1.9. Dashed lines show fits to 1/3-power-law envelope.



Faraday's law with typical electric field E on large dipolar scale and Ohm's law with $r_m = \mu\sigma Vd >> 1$ give us

$$VB / H = E / d, \qquad E = sVB. \qquad (5, 6)$$

Neglecting by magnetic diffusivity and using $d>>h$ in the induction equation I estimate the inverse time of magnetic field change as

$$sV / h = Vd / Hh. \qquad (7)$$

Supposing that the work of Archimedean force is of the order of magnetic energy time-change I estimate

$$B^2 Vd / Hh\mu = \rho AV \qquad (8)$$

The equations (1-8) give the first principles' scaling law known previously only from compilation of many numerical simulations:

$$B = (\mu\rho)^{1/2} (FH)^{1/3} \qquad (9)$$

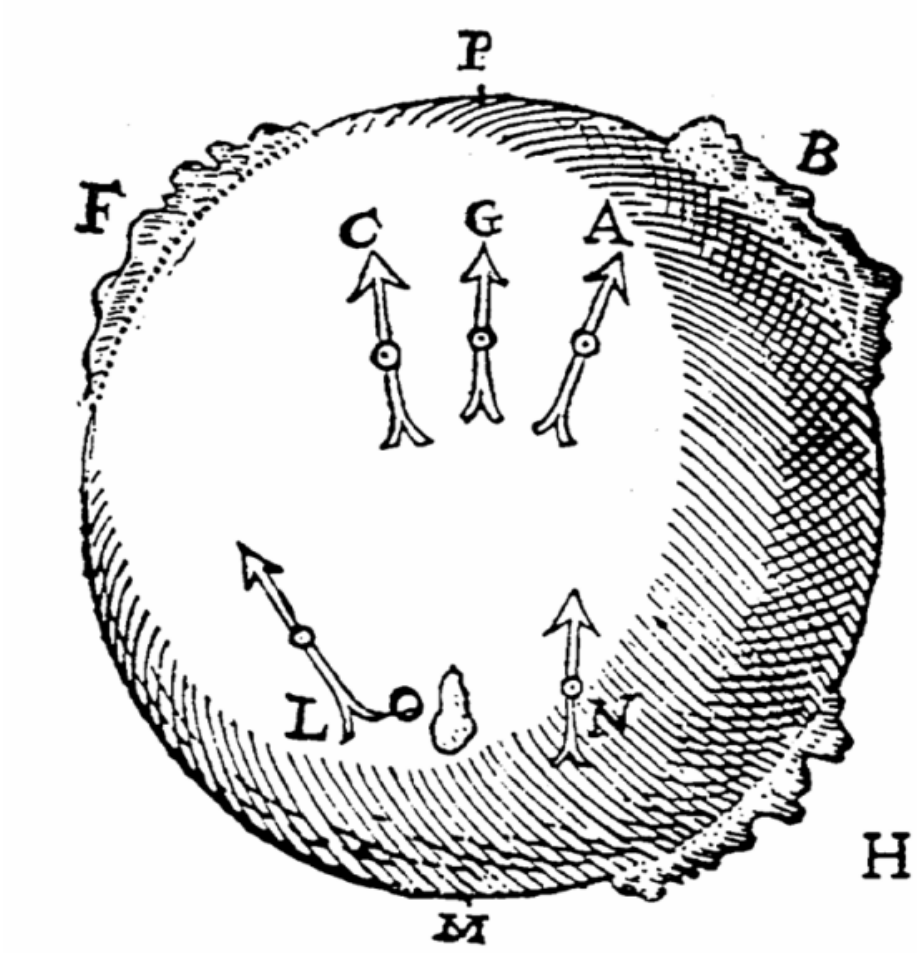
Value, unit	Mars	Earth	Jupiter	Saturn
F, 10^{-13} m^2/s^3	4	2	200	100
Ω, 10^{-5}/s	7.3	7.3	17.7	16.4
ρ, Mg/m^3	10	11	1.8	1.8
H, Mm	1.1	2.3	41	16
B, mT	1	2	5	3
h, km	4	6	45	25
d, km	30	90	500	250
V, mm/s	1.2	1	9	6
$s=d/H$	0.03	0.04	0.01	0.02
$r_m=\mu\sigma Vd$	300	700	350	120

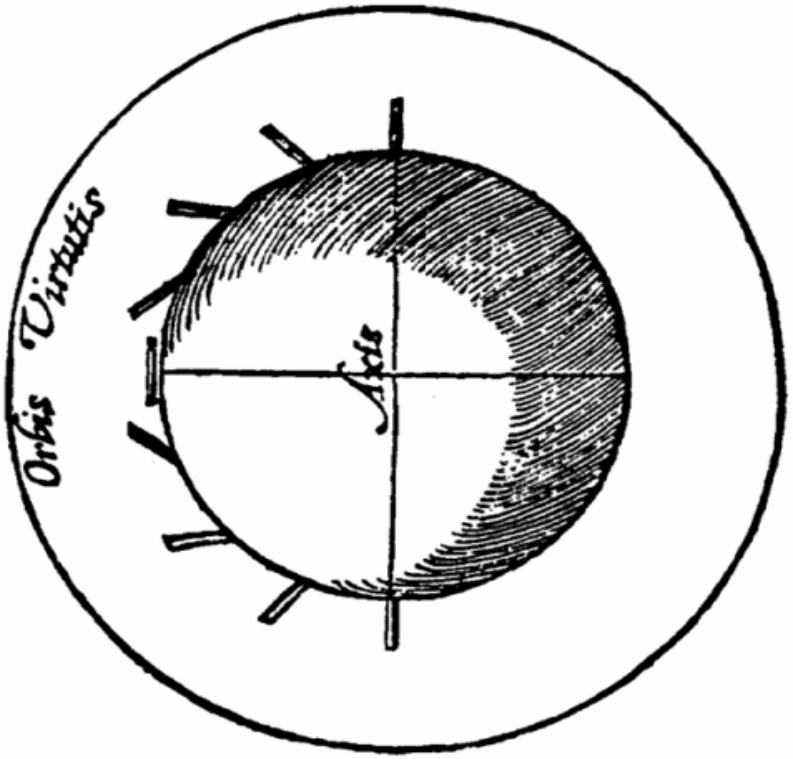
For r_m~1 magnetic energy could be of order kinetic one or $d=h$ and

$$B = (\mu\rho)^{1/2} V \quad (10)$$

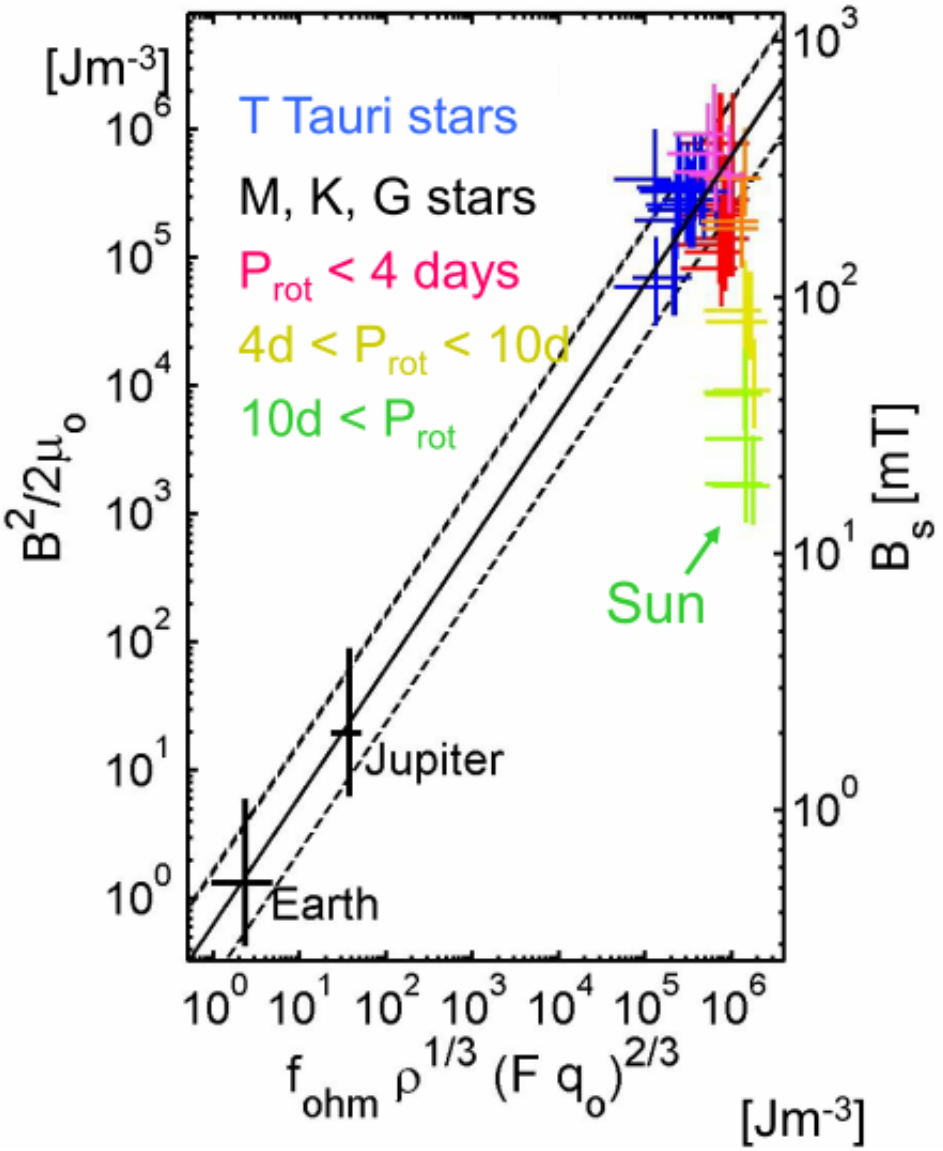
Value, unit	Ganymede	Uranus	Neptune
F, 10^{-14} m^2/s^3	6	3000	5000
Ω, 10^{-5}/s	10	10	11
ρ, Mg/m^3	8	2.5	3
H, Mm	0.5	4	5
B, μT	15	500	700
$h=d$, km	1	20	25
V, mm/s	0.2	8	12
$r_m=\mu\sigma V d$	0.2	1	1.5

Спасибо за внимание





Comparison with planets and stars



The observed fields of rapidly rotating low-mass stars agree with the prediction as well as that of Jupiter and Earth

⇒ confirmation for scaling law

⇒ dynamos in planets and (some) stars may be similar

Christensen et al, Nature, 2009