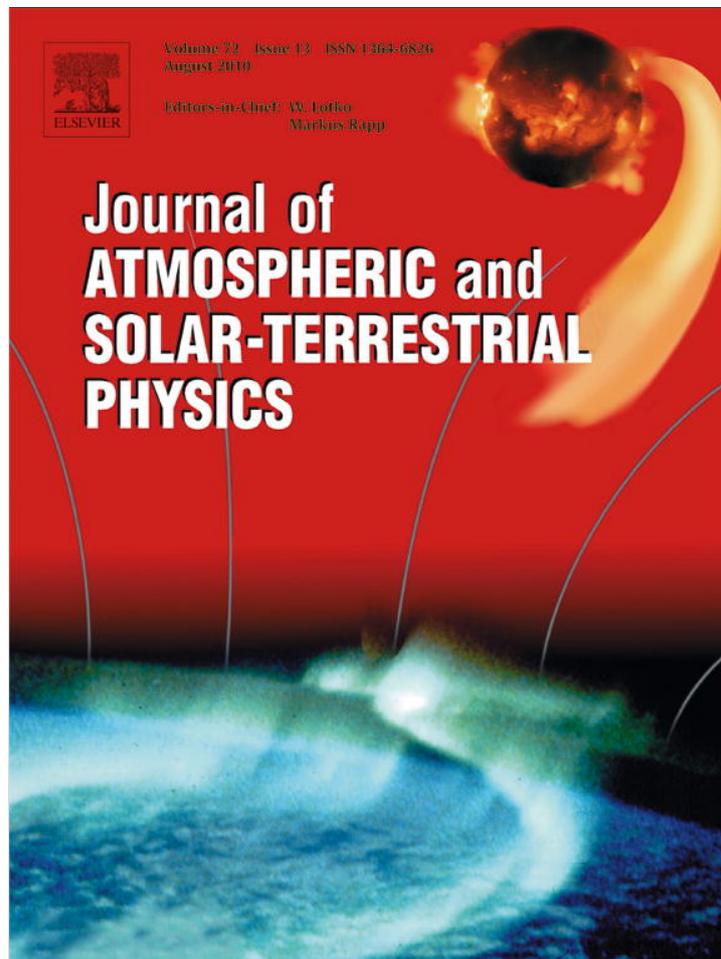


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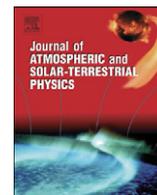
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## Generation of internal gravity vortices in the high-latitude ionosphere

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## ABSTRACT

It is shown that ionosphere heating by DC electric field leads to instability of acoustic-gravity waves and to the formation of solitary internal gravity vortex structures. These dipole type vortices with characteristic transverse size of the order of several kilometers are propagated in the lower ionosphere with subsonic velocity. The threshold values of the electric field needed to suppress the wave damping caused by the interaction of induced current with the geomagnetic field and to provide the vortex generation are found. The considered physical mechanism is applicable to the generation of internal gravity vortices and related ionospheric disturbances when the ionosphere is influenced by the electric field of seismic origin exceeding the threshold value.

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## 1. Introduction

Numerous publications indicate the growing interest in acoustic-gravity waves (AGW) in the ionosphere due to a role these waves can play in the dynamics of ionospheric plasma and their possible applications for explanation of some disturbances produced in the ionosphere by thunderstorms, earthquakes, volcano eruptions, typhoons, etc. (Hines, 1968; Hooke, 1968; Hines and Hooke, 1970; Rottger, 1981; Kim and Mahrt, 1992; Igarashi et al., 1994; Aburdzhaniya, 1996; Kaladze, 1998; Sorokin et al., 1998, 2005a, b; Chmyrev et al., 1999; Sorokin and Chmyrev, 1999; Kanamori, 2004; Kaladze et al., 2007, 2008a, b). The nonlinear acoustic-gravity wave propagation was methodically investigated by Stenflo (1987, 1990, 1996), Stenflo and Stepanyants (1995), Kaladze and Tsamalashvili (1997) and Kaladze (1998) who have considered the localized solutions in the form of dipolar vortices. Other types of nonlinear AGW structures such as the vortex chains, the tripolar and the axially symmetric monopole vortices and the structures in the form of a row of counter-rotating vortices were constructed by Stenflo (1994), Jovanovic et al. (2001, 2002) and Pokhotelov et al. (2001). The most significant results on the nonlinear acoustic-gravity waves are summarized in the review paper by Stenflo and Shukla (2009).

The theory of dissipative instability of acoustic-gravity waves has been proposed for interpretation of experimental satellite data on the small scale plasma density and transverse magnetic field variations in the ionosphere over a seismically active zone by

Sorokin et al. (1998), Chmyrev et al. (1999) and Sorokin and Chmyrev (1999). A similar physical mechanism was suggested for the formation of VLF ducts in seismically disturbed ionosphere (Sorokin et al., 2000). The instability is driven by DC electric field, which arises in the ionosphere during the enhancement of seismic activity (Chmyrev et al., 1989; Gusheva et al., 2008, 2009). The theory of this seismic induced DC electric field in the ionosphere is presented in Sorokin et al. (2001, 2005a, b, 2006).

Aburdzhaniya (1996) has considered the formation of AGW vortices in the convectively unstable ionosphere (Brunt-Vaisala frequency  $\omega_g^2 < 0$ ) and suggested the mechanism for the intensification of atomic oxygen emission (557.7 nm) due to enhancement of neutral gas density within the vortex structure. The applicability of the result to real ionosphere seems doubtful because the case when altitudinal plasma temperature gradient exceeds the gradient of plasma density is rather exotic and does not correspond to existing ionosphere models.

Kaladze et al. (2008b) have derived the equations describing the nonlinear solitary inertio-gravity vortices in stable stratified ( $\omega_g^2 > 0$ ) ionosphere taking into account the interaction of induced ionospheric current with the geomagnetic field. It was shown that in the E- and F- layers of the ionosphere these waves decay with the damping rate of the same order of magnitude as in the linear case.

Our paper presents further generalization of the nonlinear equations (Kaladze et al., 2008b) for the exponentially inhomogeneous ionosphere with finite magnitude DC electric field. As shown below, the Joule heating by the electric field leads to instability and to the formation of dipolar internal gravity vortices in the upper E-layer and the F-layer. This process occurs when DC electric field magnitude exceeds the threshold value defined by

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the damping rate in the absence of the electric field. Thus the inclusion of DC electric field allows eliminating the damping effect and the destruction of vortices launched into the ionosphere from the external sources and also provides the generation of solitary vortex structures directly in the lower ionosphere.

## 2. Basic equations for internal-gravity waves

We consider the internal gravity wave (IGW) propagation in quasi-neutral weakly ionized plasma of the lower ionosphere consisting of electrons, ions and neutral particles immersed in the geomagnetic field  $\mathbf{B}$  and influenced by DC electric field  $\mathbf{E}$ . The basic idea of this consideration is to find a balance between the wave damping caused by the interaction of induced current with the geomagnetic field and the wave growth due to dissipative IGW instability in external electric field. In linear approximation this idea was verified by Sorokin et al. (1999, 2000) who have found the threshold values of the electric field needed to suppress the attenuation and to provide the wave growth.

To describe the properties of the nonlinear waves we use the momentum equation, the equation of state and the generalized Ohm's law:

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla P + \rho \mathbf{g} + \frac{1}{c} [\mathbf{j} \times \mathbf{B}], \quad (1)$$

$$\frac{\rho^\gamma}{\gamma-1} \frac{d}{dt} \left( \frac{P}{\rho^\gamma} \right) = Q, \quad (2)$$

$$\mathbf{j} = \sigma_{\parallel} \mathbf{E}_{\parallel} + \sigma_p \mathbf{E}_{\perp} + \sigma_h (\mathbf{B} \times \mathbf{E}'_{\perp}) / B, \quad (3)$$

$$\mathbf{E}'_{\perp} = \mathbf{E} + \frac{1}{c} (\mathbf{v} \times \mathbf{B}),$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla,$$

where  $\mathbf{v}$ ,  $\rho$  and  $P$  are the mean-mass velocity characterizing the motion of gas as a whole, its density and pressure, respectively,  $\mathbf{g}$  is the gravitational acceleration,  $c$  is the light speed,  $\gamma = c_p/c_v$  is a ratio of the heat capacities at constant pressure and volume,  $E_{\parallel}$  is the geomagnetic field-aligned electric field,  $Q$  describes a source of energy, which is the Joule heating ( $\mathbf{j} \cdot \mathbf{E}$ ) in our case,  $\sigma_{\parallel}$ ,  $\sigma_p$  and  $\sigma_h$  are the parallel (field-aligned), Pedersen, and Hall conductivities, respectively:

$$\sigma_{\parallel} = e^2 N \left( \frac{1}{mv_e} + \frac{1}{Mv_{in}} \right),$$

$$\sigma_p = e^2 N \left[ \frac{v_e}{m(\omega_e^2 + v_e^2)} + \frac{v_{in}}{M(\omega_i^2 + v_{in}^2)} \right],$$

$$\sigma_h = e^2 N \left[ \frac{\omega_e}{m(\omega_e^2 + v_e^2)} - \frac{\omega_i}{M(\omega_i^2 + v_{in}^2)} \right],$$

where  $e$  is electron charge,  $N$  is electron number density,  $v_e = v_{ei} + v_{en}$ , where  $v_{ei}$  and  $v_{en}$  are the effective collision frequencies of electrons with ions and neutrals,  $v_{in}$  is the collision frequency of ions with neutrals,  $\omega_e$ ,  $\omega_i$  are the cyclotron frequencies of electrons and ions and  $m$ ,  $M$  are electron and ion masses. We assume  $\mathbf{E}_{\parallel} = 0$  since  $\sigma_{\parallel} > 10^4 \sigma_p$  in the F- and upper E-layers of the ionosphere. Taking into account the fact that in these regions  $v_e \ll \omega_e$ ,  $v_{in} \ll \omega_i$  and  $(mv_e/Mv_{in}) \ll 1$  we can use the simplified expressions for the conductivities as follows:

$$\sigma_p \simeq e^2 N \frac{v_{in}}{M\omega_i^2}, \quad \sigma_h \simeq \left( \frac{v_{in}}{\omega_i} \right) \sigma_p \ll \sigma_p,$$

where  $v_{in} = qv_i^* N_n$ ,  $v_i^* = (8RT/\pi)^{1/2}$ ,  $q$  is the cross-section of ion scattering on molecules,  $N_n$  is the number density of molecules,  $T$

is gas temperature,  $R = c_p - c_v$  is the universal gas constant and  $v_i^*$  is mean thermal velocity of ions. Thus the dependence of  $\sigma_p$  on thermodynamical quantities can be presented as

$$\sigma_p = \mu \rho_i \rho(T)^{1/2}; \quad \mu = 8q^2 c^2 / \pi M B^2. \quad (4)$$

Let us introduce the right hand Cartesian coordinate system  $(x, y, z)$  with the  $x$ -axis directed from the west to the east, the  $y$ -axis from the south to the north, and the  $z$ -axis upward along the local vertical. We assume that the magnetic field  $\mathbf{B}$  is vertical and downward and therefore the consideration is applicable to high-latitude ionosphere of the northern hemisphere. DC electric field  $\mathbf{E}$  is directed along the  $x$ -axis. Let us consider the behavior of small perturbations of the velocity  $\mathbf{v}_1$ , density  $\rho_1$  and pressure  $P_1$  at the background of their stationary values  $\mathbf{v}_0$ ,  $\rho_0$ ,  $P_0$  and the stationary temperature  $T_0$  in the exponentially irregular ionosphere:

$$\rho_0 \sim P_0 \sim \exp(-z/H); \quad H = RT_0/g. \quad (5)$$

Ion density varies with altitude substantially slower than the neutral atmosphere density  $\rho_0$  with the scale  $H$ ; therefore it could be assumed to be constant. The estimates made in Sorokin et al. (1998, 1999) show that we can neglect the vertical velocity  $v_{z0}$  and assume the ionosphere to be at rest. In Eqs. (1)–(3)  $\mathbf{j} = \mathbf{j}_1 = \sigma_{p1} \mathbf{E} + \sigma_{p0} (\mathbf{v}_1 \times \mathbf{B}) / c$  is the current density perturbation connected with a perturbation of the ionosphere conductivity  $\sigma_{p1}$ . Using Eq. (4), the equation of state and the equality  $\rho_i/\rho_0 = \alpha(\rho_1/\rho_0)$ , one obtains (Sorokin et al., 1999)

$$\sigma_{p1} \simeq (\sigma_{p0}/2) P_1/P_0 + [(2\alpha + 1)\sigma_{p0}/2] \rho_1/\rho_0 \quad (6)$$

Finally we make use of the assumption that the perturbations of pressure are relatively small  $P_1/P_0 \ll \rho_1/\rho_0$  and the medium is incompressible ( $\nabla \cdot \mathbf{v} = 0$ ). Thus, we study the low-frequency branch of AGW—the internal gravity wave (IGW) mode. Assuming  $\partial/\partial y = 0$  we consider two-dimensional plasma motion in the  $(x, z)$  plane. With these assumptions one can introduce the stream function  $\psi$

$$v_{1x} = -\frac{\partial \psi}{\partial z}, \quad v_{1z} = \frac{\partial \psi}{\partial x} \quad (7)$$

Using Eqs. (3), (6) and (7) Eqs. (1) and (2) yield

$$\rho_0 \left[ \frac{\partial}{\partial t} (\nabla^2 \psi) + J(\psi, \nabla^2 \psi) \right] = -g \frac{\partial \rho}{\partial x} - \frac{\partial \rho_0}{\partial z} \left[ \frac{\partial}{\partial t} \left( \frac{\partial \psi}{\partial z} \right) + J \left( \psi, \frac{\partial \psi}{\partial z} \right) \right] - \frac{\sigma_{p0} B^2}{c^2} \frac{\partial^2 \psi}{\partial z^2} - \frac{\partial}{\partial z} \left( \frac{\sigma_{p0} B^2}{c^2} \right) \frac{\partial \psi}{\partial z}, \quad (8)$$

$$\frac{\partial \rho_1}{\partial t} + J(\psi, \rho_1) = \omega_1 \rho_1 + \rho_0 \left( \frac{1}{\gamma P_0} \frac{\partial P_0}{\partial z} - \frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} \right) \frac{\partial \psi}{\partial x}, \quad (9)$$

where  $J(a, b) = (\partial a/\partial x)(\partial b/\partial z) - (\partial a/\partial z)(\partial b/\partial x)$ ,  $\nabla^2 \psi = \partial^2 \psi/\partial x^2 + \partial^2 \psi/\partial z^2$ ,

$$\omega_1 = \frac{(2\alpha + 1)(\gamma - 1)}{2c_s^2 \rho_0} \sigma_{p0} E^2 \quad (10)$$

and  $c_s$  is a sound velocity. Taking into account distribution (5) we introduce

$$\rho_2 = \rho_1 \exp(z/2H), \quad \bar{\psi}_1 = \psi \exp(-z/2H), \quad \rho_0 = \rho_s \exp(-z/H), \quad \rho_s = \text{Const.} \quad (11)$$

Substitution of Eqs.(5) and (11) in Eqs. (8) and (9) yields

$$\frac{\partial}{\partial t} \left( \nabla^2 \bar{\psi}_1 - \frac{1}{4H^2} \bar{\psi}_1 \right) + J(\bar{\psi}_1, \nabla^2 \bar{\psi}_1) \exp(z/2H) = -\frac{g}{\rho_s} \frac{\partial \rho_2}{\partial x} - \omega_m \left( \frac{\partial^2 \bar{\psi}_1}{\partial z^2} - \frac{1}{4H^2} \bar{\psi}_1 \right), \quad (12)$$

$$\frac{\partial \rho_2}{\partial t} + J(\bar{\psi}_1, \rho_2) \exp(z/2H) = \omega_1 \rho_2 + \rho_s \frac{\gamma - 1}{\gamma H} \frac{\partial \bar{\psi}_1}{\partial x}, \quad (13)$$

where  $\omega_m = \sigma_{p0} B^2 / c^2 \rho_s$ . The set of Eqs. (12) and (13) looks similar to Eqs. (57) and (58) of Kaladze et al. (2008b). Substantial difference is in a term  $\omega_1 \rho_2$  in the right side of Eq. (13), which describes the Joule heating effect on the electrical conductivity and provides the wave growth as shown below. When obtaining Eqs.(12) and (13) we assumed that for the considered wave disturbances  $\partial/\partial z \gg 1/H$  and therefore one could neglect the nonlinear terms proportional to  $1/H$ . Besides we used the equality  $\partial P_0 / \partial z \simeq -g \rho_0$  according to Eq. (1) in zero approximation since  $v_{z0} \ll (gH)^{1/2}$ .

### 3. The dispersion relation and the energy dynamic low

In linear approximation, Eqs. (12) and (13) allow us to obtain the dispersion equation for IGW. Assuming  $\psi_1 \sim \rho_2 \sim \exp(-i\omega + ik_x x + ik_z z)$  one obtains

$$\omega = \left[ \frac{1}{4} \left( \omega_1 - \omega_m \frac{k_z^2 + 1/4H^2}{k^2 + 1/4H^2} \right)^2 + \frac{\omega_g^2 k_x^2 - \omega_1 \omega_m (k_z^2 + 1/4H^2)}{k^2 + 1/4H^2} \right]^{1/2} + \frac{i}{2} \left( \omega_1 - \omega_m \frac{k_z^2 + 1/4H^2}{k^2 + 1/4H^2} \right), \quad (14)$$

where  $\omega_g = [g(\gamma - 1)/\gamma H]^{1/2}$  is the Brunt-Vaisala frequency. For  $\omega_g^2 \gg \omega_1 \omega_m$  Eq. (14) takes the following form:

$$\omega \simeq \omega_g \frac{k_x}{(k^2 + 1/4H^2)^{1/2}} + \frac{i}{2} \left( \omega_1 - \omega_m \frac{k_z^2 + 1/4H^2}{k^2 + 1/4H^2} \right). \quad (15)$$

In the absence of DC electric field ( $\omega_1 = 0$ ) Eq. (15) describes the same dispersion relation and the damping rate for IGW as in Kaladze et al. (2008b). An appearance of strong enough electric field as seen from (15) suppresses the damping and provides the wave growth when

$$\omega_1 > \omega_m \frac{k_z^2 + 1/4H^2}{k^2 + 1/4H^2}.$$

This inequality defines the threshold value  $E_{th}$  of the electric field for the IGW instability as follows:

$$E_{th} = \frac{c_s}{c} B \left[ \frac{2}{(2\alpha + 1)(\gamma - 1)} \right]^{1/2}. \quad (16)$$

To find the energy dynamic low for the considered waves let us multiply Eq. (12) by  $-\psi_1$  and Eq. (13) by  $\rho_2$  and then integrate these equations over  $x$  and  $z$ . As a result one obtains

$$\frac{\partial W}{\partial t} = -\omega_m \int \left[ \left( \frac{\partial \psi_1}{\partial z} \right)^2 + \frac{\psi_1^2}{4H^2} \right] dx dz + \omega_1 \int (g^2 / \rho_s^2 \omega_g^2) \rho_2 dx dz, \quad (17)$$

where  $W$  is the energy of the wave structure:

$$W = \int \left[ \frac{1}{2} (\nabla \psi_1)^2 + \frac{\psi_1^2}{8H^2} + \frac{1}{2} (g^2 / \rho_s^2 \omega_g^2) \rho_2^2 \right] dx dz.$$

Thus, we see from Eq. (17) that the wave energy decreases if  $\omega_1 = 0$  and increases at a sufficiently large value of  $\omega_1$  corresponding to the over threshold electric field.

### 4. IGW vortex solution

We look for stationary solution of Eqs. (12) and (13) in the reference frame moving with constant velocity  $u$  along the  $x$  direction, which is dependent only on the coordinates  $\xi = x - ut$  and  $z$ . In this reference frame Eqs. (12) and (13) take the

following form:

$$-u \frac{\partial}{\partial \xi} \left( \nabla^2 \psi_1 - \frac{1}{4H^2} \psi_1 \right) + J(\psi_1, \nabla^2 \psi_1) \exp(z/2H) = -\frac{g}{\rho_s} \frac{\partial \rho_2}{\partial \xi} - \omega_m \left( \frac{\partial^2 \psi_1}{\partial z^2} - \frac{1}{4H^2} \psi_1 \right), \quad (18)$$

$$-u \frac{\partial \rho_2}{\partial \xi} + J(\psi_1, \rho_2) \exp(z/2H) = \omega_1 \rho_2 + \rho_s \frac{\gamma - 1}{\gamma H} \frac{\partial \psi_1}{\partial \xi}. \quad (19)$$

Let us look for the solution that satisfies the equation

$$\nabla^2 \psi_1 + \frac{g}{u \rho_s} \rho_2 = 0 \quad (20)$$

Substituting  $\partial \rho_2 / \partial \xi$  from Eq. (19) in Eq. (18) and using (20) we obtain

$$\frac{\partial}{\partial \xi} \left[ \nabla^2 \psi_1 + \left( \kappa^2 - \frac{1}{4H^2} \right) \psi_1 \right] = \kappa_1 \nabla^2 \psi_1 + \kappa_m \left( \frac{\partial^2 \psi_1}{\partial z^2} - \frac{1}{4H^2} \psi_1 \right), \quad (21)$$

where  $\kappa^2 = \omega_g^2 / u^2$ ,  $\kappa_1 = \omega_1 / u$  and  $\kappa_m = \omega_m / u$ . Let us consider first the event of relatively small attenuation ( $\kappa_m \ll \kappa_1$ ) as follows:

$$\frac{\partial}{\partial \xi} \left[ \nabla^2 \psi_1 + \left( \kappa^2 - \frac{1}{4H^2} \right) \psi_1 \right] = \kappa_1 \nabla^2 \psi_1.$$

Multiplying this equation by  $\exp(-pt)$  and integrating over  $\xi$  from 0 to  $\infty$  yields

$$\nabla^2 \psi_1 + \left( \kappa^2 - \frac{1}{4H^2} \right) \frac{p}{(p - \kappa_1)} \psi_1 = \frac{R_1(\xi = 0, z)}{(p - \kappa_1)}, \quad (22)$$

$$R_1(\xi = 0, z) = \nabla^2 \psi_1(\xi = 0, z) + \left( \kappa^2 - \frac{1}{4H^2} \right) \psi_1(\xi = 0, z),$$

where  $p$  is the Laplace variable. The simplest partial solution of the inhomogeneous equation (22) is  $\psi_1 = p^{-1} a_1 z (\kappa^2 - 1/4H^2)^{-1}$ , which yields the Laplace inversion

$$\psi_1 = \frac{a_1 z}{\left( \kappa^2 - \frac{1}{4H^2} \right)} \quad (23)$$

Let us rewrite the homogeneous equation (22) in the polar coordinates  $r = (\xi^2 + z^2)^{1/2}$ ,  $\theta = \arctan \xi / z$ :

$$\frac{\partial^2 \psi_1}{\partial r^2} + \frac{1}{r} \frac{\partial \psi_1}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi_1}{\partial \theta^2} + \left( \kappa^2 - \frac{1}{4H^2} \right) \frac{p}{(p - \kappa_1)} \psi_1 = 0. \quad (24)$$

The solution of this equation at  $\kappa^2 > 1/4H^2$  is

$$\psi_1(r, \theta) = a(p) \cos \theta J_1 \left[ r \left( \kappa^2 - \frac{1}{4H^2} \right)^{1/2} \left( \frac{p}{p - \kappa_1} \right)^{1/2} \right], \quad (25)$$

where  $J_1(\alpha r)$  is the Bessel function of first order and  $a(p)$  is the arbitrary function of  $p$ . To provide  $\psi_1 \rightarrow 0$  at  $p \rightarrow \infty$  we assume  $a(p) = a_0 / p$  ( $a_0 = Const$ ). Let us consider the behavior of this solution at small  $\xi$ :

$$\lim_{\xi \rightarrow 0} \psi_1(\xi) = \lim_{p \rightarrow \infty} p \psi_1(p).$$

Expansion of the Bessel function in Eq. (25) in series on small  $\delta = \kappa_1 / p$  and the Laplace inversion of two first terms of the expansion transform solution (25) into the following form:

$$\psi_1(\xi, z) \simeq a_0 \cos \theta \left[ J_1(\beta r) + \frac{\kappa_1}{2} \beta r J_1'(\beta r) \int_0^\xi e^{\kappa_1(\tau + \xi)/2} I_0 \left[ \frac{\kappa_1}{2} (\tau - \xi) \right] d\tau, \quad (26)$$

where  $\beta = (\kappa^2 - 1/4H^2)^{1/2}$ ,  $J_1'(\beta r)$  is a derivative of  $J_1(\beta r)$  and  $I_0$  is the modified Bessel function of the first kind.

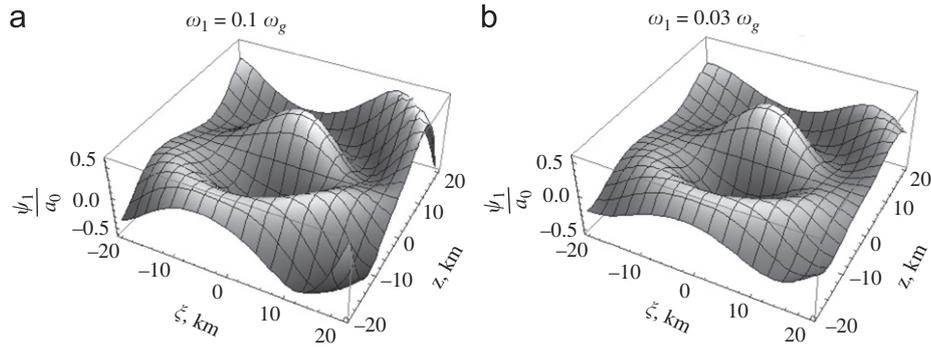


Fig. 1. The spatial distribution of  $\psi_1(\xi, z)$  function at various values of  $\omega_1$ . The following parameters have been selected:  $u=50$  m/s;  $H=10^4$  m;  $a_1=0$ .

To find the solution at large  $\xi$  let us return to Eq. (24) at  $p \rightarrow 0$ . In this case we obtain

$$\lim_{\xi \rightarrow \infty} \psi_1 = b \cos \theta K_1 \left( \beta r \sqrt{\frac{p}{\kappa_1}} \right),$$

where  $K_1$  is the modified Bessel function of the second kind and  $b=b(p)$  is the arbitrary function of  $p$ . For simplicity let us put  $b(p)=b_0 p^{-1/2}$  and  $b_0=Const$ . The Laplace inversion of the above expression yields the solution at  $\xi \rightarrow \infty$ :

$$\psi_1(\xi, z) \simeq \frac{\sqrt{\kappa_1} b_0 \cos \theta}{\beta r} \exp \left( -\frac{\beta^2 r^2}{4 \kappa_1 \xi} \right). \quad (27)$$

Combining Eq. (26) with solution (23) of the inhomogeneous equation (22) we finally obtain the general solution for small  $\xi$  as follows:

$$\psi_1(\xi, z) \simeq a_0 \cos \theta \left[ J_1(\beta r) + \frac{k_1}{2} \beta r J_1(\beta r) \int_0^\xi e^{k_1(\tau+\xi)/2} I_0 \left[ \frac{k_1}{2}(\tau-\xi) \right] d\tau \right] + \frac{a_1}{\beta} r \cos \theta. \quad (28)$$

Fig. 1 shows the plot of  $\psi_1$  as a function of coordinates  $(\xi, z)$  calculated with the help of Eq. (28). In the absence of DC electric field ( $\kappa_1=0$ ) Eq. (28) has a classical form as follows:

$$\psi_1 = a_0 \cos \theta J_1(\beta r) + (a_1/\beta) r \cos \theta.$$

Thus we obtained the localized solitary dipolar vortex, which has form (27) at large distances  $\xi \rightarrow \infty$  and form (28) at small  $\xi \rightarrow 0$ . Exact vortical solution in all ranges of  $0 < \xi < \infty$  could be found from the Laplace inversion applied to Eq. (25).

## 5. Discussion

Electric field driven generation of solitary IGW dipolar vortical structures can produce significant effects in the upper ionosphere. Dual plasma density perturbations within the structures and the corresponding alteration of electric conductivity induce the polarization current and related electric field disturbances in the upper E-layer. Because of high conductivity along the magnetic field lines these disturbances are transferred to the high altitude ionosphere and the magnetosphere by magnetic field-aligned currents, which are closed in the E-layer due to Pedersen conductivity. Since the field-aligned currents are carried by electrons, while the transverse currents are carried by ions, the upward propagation of the electric field disturbances is followed by the local variations of plasma density and the formation of plasma layers stretched along the geomagnetic field. Thus the generation of IGW vortices in the upper E-layer can induce the localized plasma density inhomogeneities and the geomagnetic field-aligned currents in the high altitude ionosphere traveling along the  $x$  (west–east) direction with constant velocity  $u$ , which is several times lower than the sound velocity in the E-layer.

The characteristic transverse size of these structures in the upper ionosphere corresponds to the vortex cross-section and is of the order of several kilometers. This mechanism was first considered in a frame of the linear theory of dissipative acoustic-gravity wave instability by Sorokin et al. (1998) and was applied to interpretation of some experimental data on the seismic related ionospheric disturbances by Chmyrev et al. (1999), Sorokin and Chmyrev (1999) and Sorokin et al. (2000). The nonlinear localized IGW solutions obtained in the present paper can be used for further development of the theory of seismic impact on the ionosphere.

Critical point of consideration in both the linear and nonlinear approaches is the magnitude of DC electric field needed for wave excitation. We already found the threshold value of the electric field required to suppress the wave damping caused by the interaction of induced current with the geomagnetic field and to provide the vortex generation. Let us estimate this value for the high-latitude ionosphere. Assuming  $\gamma=1.4$ ,  $\alpha=2$ ,  $B=5 \times 10^4$  nT and  $c_s=300$  m/s we obtain from Eq. (16)  $E_{th} \simeq 15$  mV/m. This is a rather common value for the auroral ionosphere.

Besides the electric field there are some other sources of influencing the ionosphere, which could produce similar effect on IGW generation. Among them are large amplitude electromagnetic waves radiated by the ground-based HF and VLF transmitters. The long-term operation of these “heaters” can lead to excitation of IGW and the formation of solitary dipolar structures in the ionosphere over the heating facilities. As associated effect one can expect the generation of localized plasma density inhomogeneities stretched along the geomagnetic field lines, which work as the VLF ducts.

## 6. Conclusion

In this paper we have investigated the influence of DC electric field on nonlinear internal gravity wave propagation in the stable stratified ionosphere. The nonlinear governing equations are deduced, which take into account both the wave decay caused by the interaction of the induced ionospheric current with the geomagnetic field and the wave growth due to dissipative IGW instability in the electric field. It is shown that the Joule heating by the electric field leads to the formation of solitary dipolar internal gravity vortices in the upper E- and F-layers. This process occurs when DC electric field magnitude exceeds the threshold value defined by the damping rate in the absence of the electric field. An estimate made for the auroral ionosphere gives the value  $E_{th} \simeq 15$  mV/m. Thus the inclusion of DC electric field allows one to eliminate the damping effect and the destruction of vortices launched into the ionosphere from the external sources and provides the generation of solitary vortex structures directly in the ionosphere.

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